

**Lecture Series 2000-06**

**Theoretical and Experimental Modelling of  
Particulate Flows**

**April 3 – 7, 2000**

**von Karman Institute for Fluid Dynamics**

**Overview and Fundamentals  
Part I and II**

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## Summary

The lecture summarises the fundamentals of fluid-particle flows and important physical phenomena influencing the particle motion, such as wall collisions and inter-particle collisions. The first part introduces a classification of multiphase flows and dispersed particle-laden flows and provides the definitions for describing the content of the dispersed phase. In the second chapter some problems of engineering relevance are introduced, in order to emphasise the importance of a detailed description of particle motion in turbulent flows. The equations to calculate the motion of particles are introduced in part three, including a summary of all relevant forces acting on particles. Recent results on transverse lift forces due to shear and particle rotation are also introduced. Finally, the importance of the different forces, e.g. added mass, Basset force, and pressure force, for calculating the particle motion in an oscillatory flow field is discussed.

Part four introduces the fundamentals for calculating the collision of spherical particles with walls and the importance of wall roughness for this process. Bases on Lagrangian calculations of the particles in channel and pipe flows, the importance of wall collisions and wall roughness for the particle behaviour is emphasised. Moreover, it is pointed out, that the modelling of wall collisions is essential for predicting the correct pressure loss in pneumatic conveying.

The fifth part is devoted to inter-particle collisions. After an introduction of the different collision mechanisms in laminar and turbulent flows, the basic equations for calculating inter-particle collisions in a Lagrangian frame are introduced. The importance of inter-particle collisions is demonstrated on the basis of Lagrangian calculations of a particle-laden horizontal channel flow. It is shown that whenever inhomogenities of the particle concentration develop in a flow, inter-particle collisions become of great importance already for quite low overall particle concentration.

The final chapter gives a brief introduction of the numerical methods being applied for the calculation of dispersed particle-laden flows.

## Content

|           |                                                           |           |
|-----------|-----------------------------------------------------------|-----------|
| <b>1.</b> | <b>Classification of Multiphase flows</b>                 | <b>4</b>  |
| <b>2.</b> | <b>Examples of Engineering Practice</b>                   | <b>7</b>  |
| <b>3.</b> | <b>Particle Motion in Fluids</b>                          | <b>10</b> |
| 3.1       | Drag Force                                                | 11        |
| 3.2       | Pressure Gradient and Buoyancy Force                      | 17        |
| 3.3       | Added Mass and Basset Force                               | 18        |
| 3.4       | Body Forces                                               | 19        |
| 3.5       | Slip-Shear Lift Force                                     | 20        |
| 3.6       | Slip-Rotation Lift Force                                  | 22        |
| 3.7       | Torque                                                    | 24        |
| 3.8       | Response Time and Stokes Number                           | 25        |
| 3.9       | Importance of the Different Forces                        | 26        |
| <b>4.</b> | <b>Particle-Wall Collisions</b>                           | <b>29</b> |
| 4.1       | Velocity Change During Wall Collisions                    | 29        |
| 4.2       | Wall Roughness Effects                                    | 31        |
| 4.3       | Importance of Wall Collisions in Channel and Pipe Flows   | 34        |
| <b>5.</b> | <b>Inter-Particle Collisions</b>                          | <b>39</b> |
| 5.1       | Importance of Inter-Particle Collisions                   | 41        |
| 5.2       | Particle Velocity Change due to Inter-Particle Collisions | 44        |
| 5.3.      | Inter-particle Collision Effects in Turbulent Flows       | 48        |
| <b>6.</b> | <b>Methods for the Prediction of Multiphase Flows</b>     | <b>54</b> |
| <b>7.</b> | <b>Nomenclature</b>                                       | <b>57</b> |
| <b>8.</b> | <b>References</b>                                         | <b>59</b> |

## 1. Classification of Multiphase flows

Multiphase flows may be encountered in various forms in industrial practice (Figure 1), as for example, transient flows with a transition from pure liquid to a vapour flow as a result of external heating, separated flows (i.e. stratified flows, slug flows, or film flows), and dispersed two-phase flows where one phase is present in the form of particles, droplets, or bubbles in a continuous carrier phase (i.e. gas or liquid). Such dispersed two-phase flows, which are the main concern of the present contribution, are encountered in numerous technical and industrial processes, as for example in particle technology, chemical engineering, and biotechnology. Dispersed two-phase flows may be classified in terms of the different phases being present as summarised in Table 1 together with some of the most important industrial processes.

Additionally, numerous processes may involve more than two phases (i.e. multiphase flows), as for example in a spray scrubber where droplets and solid particles are dispersed in a gas flow and the aim is to collect the particles by the droplets.

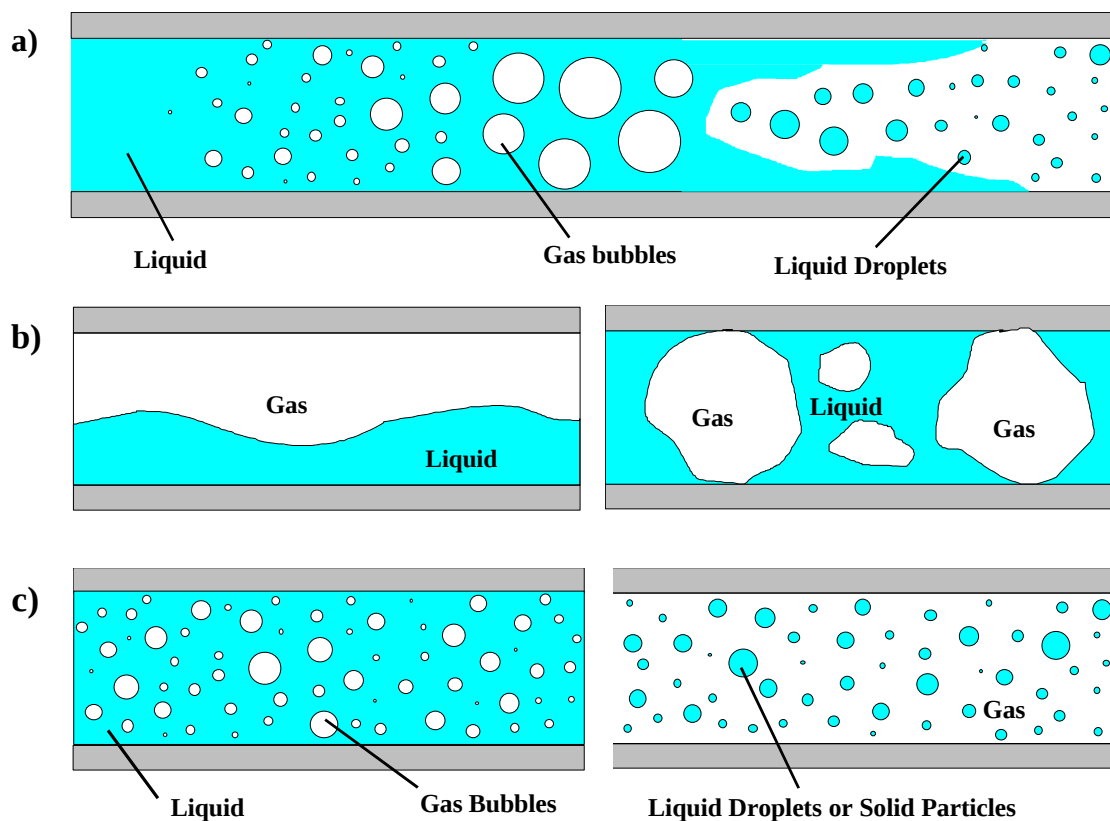


Figure 1.1: Different regimes of two-phase flows, a) transient two-phase flow, b) separated two-phase flow, c) dispersed two-phase flow

| Continuous/Dispersed Phase | Industrial/Technical Application                                                     |
|----------------------------|--------------------------------------------------------------------------------------|
| Gas-solid flows            | pneumatic conveying, particle separation in cyclones and filters, fluidised beds     |
| Liquid-solid flows         | hydraulic conveying, liquid-solid separation, particle dispersion in stirred vessels |
| Gas-droplet flows          | spray drying, spray cooling, spray painting, spray scrubbers                         |
| Liquid-droplet flows       | mixing of immiscible liquids, liquid-liquid extraction                               |
| Liquid-gas flows           | bubble columns, aeration of swage water, flotation                                   |

Table 1.1: Summary of two-phase flow systems and important industrial and technical processes.

For the characterisation of dispersed two-phase flows different properties are used, which are briefly summarised below. The volume fraction of the dispersed phase is the volume occupied by the particles in a unit volume. Hence this property is given by:

$$\alpha_p = \frac{\sum_i N_i V_{pi}}{V} \quad (1.1)$$

where  $N_i$  is the number of particles in the size fraction  $i$ , having the particle volume  $V_{pi} = \pi/6 D_{pi}^3$ . The particle diameter  $D_{pi}$  in this context is the volume equivalent diameter of a sphere. Since the sum of the volume fraction of the dispersed phase and the continuous phase is unity, the continuous volume fraction is:

$$\alpha_F = (1 - \alpha_p) \quad (1.2)$$

The bulk density or concentration of the dispersed phase is the mass of particles per unit volume and hence given by:

$$\rho_p^b = c_p = \alpha_p \rho_p \quad (1.3)$$

Correspondingly, the bulk density of the continuous phase is:

$$\rho_F^b = (1 - \alpha_p) \rho_F \quad (1.4)$$

The sum of both bulk densities is called mixture density:

$$\rho_m = \rho_F^b + \rho_p^b = (1 - \alpha_p) \rho_F + \alpha_p \rho_p \quad (1.5)$$

Often the particle concentration is also expressed by the number of particles per unit volume, as for example in clean-room technology:

$$n_p = \frac{N_p}{V} \quad (1.6)$$

Especially in gas-solid flows the mass loading is frequently used, which is defined as the mass flux of the dispersed phase to that of the fluid:

$$\eta = \frac{\alpha_p \rho_p U_p}{(1 - \alpha_p) \rho_F U_F} \quad (1.7)$$

The proximity of particles in a two-phase flow system may be estimated from the inter-particle spacing, which however can be only determined for regular arrangements of the particles. For a cubic arrangement the inter-particle spacing, i.e. the distance between the centres of particles, is obtained from:

$$\frac{L}{D_p} = \left( \frac{\pi}{6 \alpha_p} \right)^{1/3} \quad (1.8)$$

For a volume fraction of 1 % the spacing is 3.74 diameters and for 10 % only 1.74. Hence, for such high volume fractions the particles cannot be treated to move isolated, since fluid dynamic interactions become of importance. In many practical fluid-particle systems however, the particle volume fraction is much lower. Consider for example a gas-solid flow (particle density  $\rho_p = 2500 \text{ kg/m}^3$ , gas density of  $\rho_F = 1.18 \text{ kg/m}^3$ ) with a mass loading of one and assume no slip between the phases, then the volume fraction is about 0.05 % (i.e.  $\alpha_p = 5 \cdot 10^{-4}$ ). This results in an inter-particle spacing of about 10 particle diameters, hence, under such a condition a fluid dynamic interaction may be neglected.

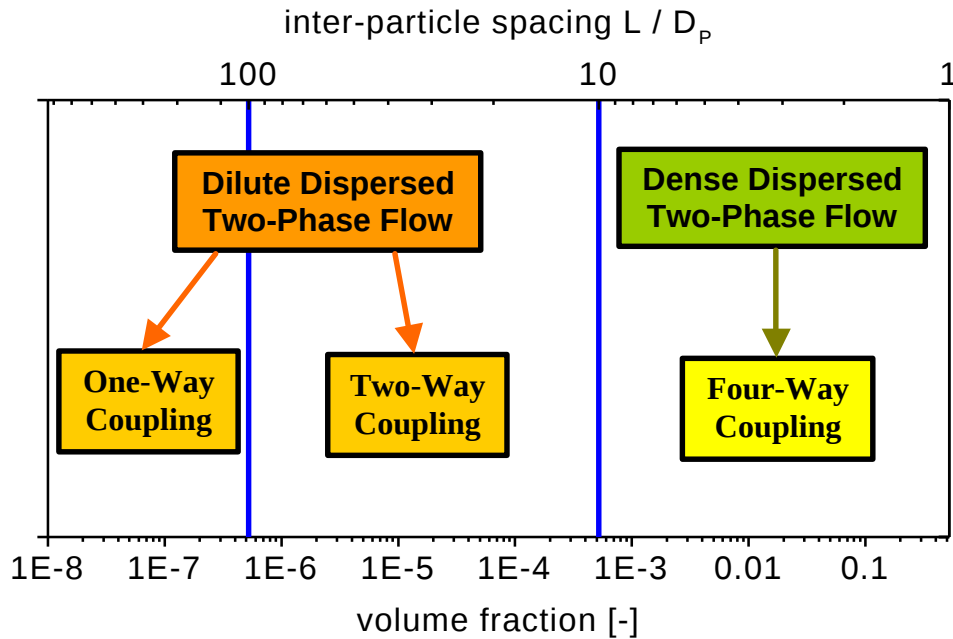


Figure 1.2 Regimes of dispersed two-phase flows as a function of particle volume fraction

A classification of dispersed two-phase flows with regard to the importance of interaction mechanisms was provided by Elghobashi (1994). Generally it is separated between dilute and dense two-phase flows (Figure 1.2). A two-phase system may be regarded as dilute for volume fractions up to  $\alpha_p = 10^{-3}$  (i.e.  $L/D_p \approx 8$ ). In this regime the influence of the particle phase on the fluid flow may be neglected for  $\alpha_p < 10^{-6}$  (i.e.  $L/D_p \approx 80$ ). For higher volume fractions the influence of the particles on the fluid flow, which is often referred to as two-way coupling, needs to be accounted for. In the dense regime (i.e. for  $\alpha_p > 10^{-3}$ ) additionally inter-particle interactions (i.e. collisions and fluid dynamic interactions between particles) become of importance. Hence, this regime is characterised by the so-called four-way coupling. Another interpretation for the separation between dilute and dense two-phase flow, which also accounts for particle inertia, will be introduced in the chapter „inter-particle collisions“.

## 2. Examples of Engineering Practice

The classical engineering approach for designing dispersed two-phase flow processes is based on empirical correlations which have been developed over the years, or on simple models involving generally crude assumptions. The minimum impeller speed for suspending particles in a stirred vessel, for example, is found from empirical correlations which are obtained from numerous experimental investigations (Liepe et al. 1998).

The average cut size of a reverse flow cyclone is obtained from a force balance for a particle by using rather crude assumptions about the flow field in the cyclone (Muschelknautz 1970). The force balance is applied on the surface of the outlet tube extended to the bottom of the cyclone (Figure 2.1). On this surface a constant radial velocity is assumed over the height. Hence the balance of radial drag force and centrifugal force yields the following cut size by assuming Stokes drag:

$$D_C = \sqrt{\frac{18 \mu_F V_{ri} r_i}{\rho_p V_{\phi i}^2}} \quad (2.1)$$

where  $r_i$  is the radius of the outlet pipe,  $V_{ri}$  is the radial velocity, and  $V_{\phi i}$  is the tangential velocity. The assumption of a constant radial velocity is justified in most cases, hence it may be calculated from the volume flow rate  $\dot{V}$  and the surface of the exit tube extension with the height  $h_i$  (Figure 2.1) as:

$$V_{ri} = \frac{\dot{V}}{2 \pi r_i h_i} \quad (2.2)$$

However, the calculation of the tangential velocity of the particle is associated with several crude assumptions about the flow field. According to Muschelknautz (1970) the tangential velocity is obtained from a simple balance of the tangential momentum by accounting for inlet effects through the coefficient  $\alpha$  and wall friction by the coefficient  $\lambda$ :

$$V_{\phi i} = \frac{V_i}{\alpha F \frac{r_i}{r_e} + \lambda \frac{h}{r_i}} \quad (2.3)$$

The value  $F$  is the ratio of the cross-section of the inlet to that of the outlet tube and  $V_i$  is the mean gas velocity in the outlet tube. This simple model predicts only one cut size of the reverse flow cyclone, neglecting details of the complex flow and turbulence, which is surely important for the separation of small particles. The grade efficiency curve of the cyclone may be only obtained by additional experimental information. Recently, Frank et al. (1999) have shown that the particle separation in cyclones may be effectively calculated using the Euler/Lagrange approach. However, the results showed that one essential physical phenomenon needs to be accounted for, namely particle agglomeration. Hence the predicted grade efficiency curve was shifted to larger particles compared to the measurements.

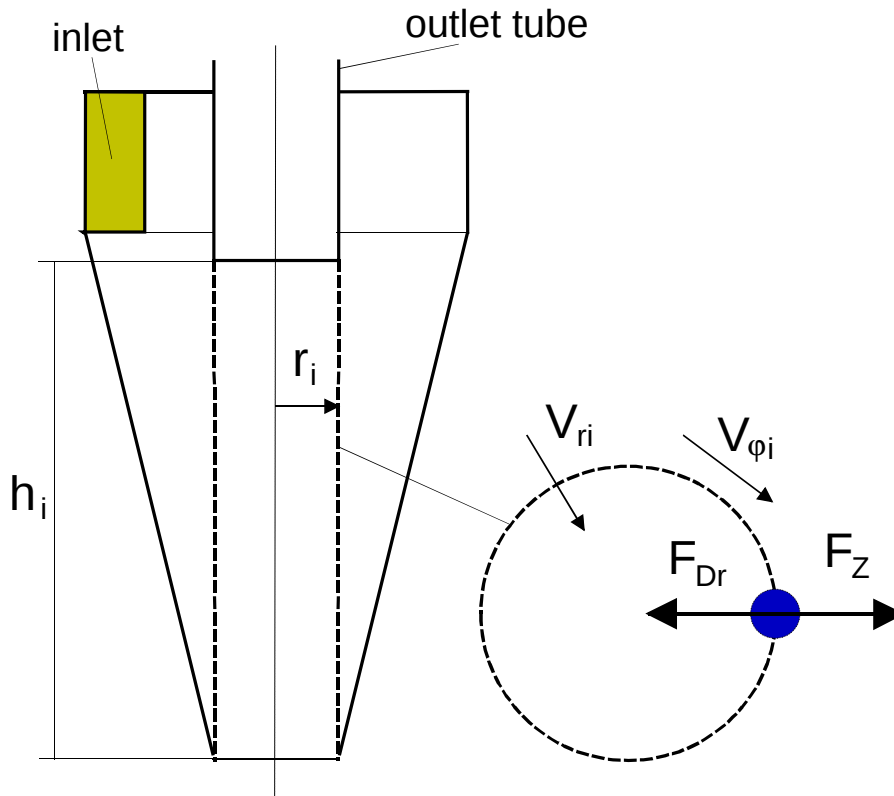


Figure 2.1: Geometry of a reverse flow cyclone and illustration of particle separation model

Another example of engineering practice is the design of spray scrubbers for particle separation. Actually, the prediction of the separation efficiency of a spray scrubber requires much information about the droplet motion in such a device. The starting point for the lay-out is the single droplet capture efficiency. Its prediction however requires already detailed information about the flow field around the droplet. Depending on the droplet Reynolds number, the single droplet capture efficiency may vary over a wide range. The limiting cases are the viscous flow assumption ( $Re_d \rightarrow 0$ ) and potential theory ( $Re_d \rightarrow \infty$ ). Turbulence of the ambient gas phase is generally neglected. According to Schuch and Löffler (1978) the single droplet capture efficiency is given by:

$$\eta = \left( \frac{\psi}{\psi + a} \right)^b \quad (2.4)$$

where  $\psi$  is the inertial parameter or Stokes number obtained from:

$$\psi = \frac{\rho_p D_p^2 U_{rel}}{18 \mu_F D_d} \quad (2.5)$$

In Eq. 2.5,  $\rho_p$  is the particle material density,  $D_p$  the particle diameter,  $D_d$  the diameter of the droplet, and  $U_{rel}$  is the local relative velocity between droplet and gas phase. The parameters  $a$  and  $b$  depend on the droplet Reynolds number (Schuch & Löffler 1978). For viscous flow (i.e.  $Re_d < 1$ ) it was found:

$$a = 0.65, \quad b = 3.7$$

and for potential flow (i.e.  $Re_d \gg 1$ ) the parameters are:

$$a = 0.25, \quad b = 2.0.$$

The scale-up from the single-droplet capture efficiency to the prediction of the grade efficiency of a spray scrubber, however, is not straightforward and requires the calculation of the droplet trajectories. In general, both the droplets and the particles to be separated have a certain size distribution, so that a simple engineering design of spray scrubber becomes impossible. In order to allow for an appropriate design, the flow field in the scrubber needs to be calculated for example using CFD (computational fluid dynamics). Subsequently a calculation of the droplet trajectories through the scrubber is required by accounting for the droplet and particle size distributions, which have an influence on the single droplet capture efficiency (see Eq. 2.4).

The examples introduced here demonstrate, that CFD may considerably support the lay-out of processes in the field of particle technology. For modelling the particle phase in a dispersed system the Lagrangian approach is most attractive since it allows a detailed consideration of the relevant physical effects influencing the particle motion. In the following chapters a

number of the relevant physical effects in dispersed two phase flows will be introduced and discussed.

### 3. Particle Motion in Fluids

The motion of particles in fluids is described in a Lagrangian way by solving a set of ordinary differential equations along the trajectory in order to calculate the change of particle location and the linear and angular components of the particle velocity. This requires the consideration of all relevant forces acting on the particle. The equation of motion for small particles in a viscous quiescent fluid (i.e. for small particle Reynolds-numbers, which is also referred to as Stokes flow) goes back to the pioneering work of Basset (1888), Boussinesq (1885) and Oseen (1927). Therefore, the equation of motion is mostly referred to as BBO-equation. Numerous publications deal with the extension of the BBO equation for turbulent flows. The thesis of Tchen (1949) was probably the first study on particle motion in turbulent flows based on the BBO equation. A rigorous derivation of the equation of motion for small particles in non-uniform flow has been performed by Maxey and Riley (1983). Neglecting the Faxen terms the equation proposed by Maxey and Riley (1983) for small particle Reynolds numbers is as follows:

$$m_p \frac{d\vec{u}_p}{dt} = \frac{18\mu_F}{\rho_p D_p^2} m_p (\vec{u}_F - \vec{u}_p) - m_F \frac{D\vec{u}_F}{Dt} + 0.5 m_F \left( \frac{D\vec{u}_F}{Dt} - \frac{d\vec{u}_p}{dt} \right) + 9 \sqrt{\frac{\rho_F \mu_F}{\pi}} \frac{m_p}{\rho_p D_p} \int_0^t \frac{D\vec{u}_F - \frac{d\vec{u}_p}{dt}}{(t-\tau)^{3/2}} d\tau + (m_p - m_F) \vec{g} \quad (3.1)$$

In the following section a possible extension of the BBO-equation for higher particle Reynolds numbers will be introduced. In addition other forces which might be relevant for certain conditions, such as for example transverse lift forces, will be introduced and their relevance will be analysed. Considering spherical particles and neglecting heat and mass transfer phenomena, the calculation of particle trajectories requires the solution of three ordinary differential equations when particle rotation is accounted for. Hence, the differential equations for calculating the particle location, and the linear and angular velocities in vector form are given by:

$$\frac{d\vec{x}_p}{dt} = \vec{u}_p \quad (3.2)$$

$$m_p \frac{d\vec{u}_p}{dt} = \sum \vec{F}_i \quad (3.3)$$

$$I_p \frac{d\vec{\omega}_p}{dt} = \vec{T} \quad (3.4)$$

where  $m_p = \pi/6 \rho_p D_p^3$  is the particle mass,  $I_p = 0.1 m_p D_p^2$  is the moment of inertia for a sphere,  $\vec{F}_i$  represents the different relevant forces acting on the particle, and  $\vec{T}$  is the torque acting on a rotating particle due to the viscous interaction with the fluid.

Analytical solutions for the different forces and the torque only are available for small particle Reynolds numbers (i.e. Stokes regime). An extension to higher Reynolds numbers is generally based on empirical correlations which are derived from experiments.

### 3.1 Drag Force

In most fluid-particle systems the drag force is dominating the particle motion and consists of a friction and form drag. The extension of the drag force to higher particle Reynolds numbers is based on the introduction of a drag coefficient  $C_D$  being defined as:

$$C_D = \frac{F_D}{\frac{\rho_F}{2} (\vec{u}_F - \vec{u}_p)^2 A_p} \quad (3.5)$$

where  $A_p = \pi/4 D_p^2$  is the cross-section of a spherical particle. The drag force is then expressed by:

$$\vec{F}_D = \frac{3}{4} \frac{\rho_F m_p}{\rho_p D_p} C_D (\vec{u}_F - \vec{u}_p) |\vec{u}_F - \vec{u}_p| \quad (3.6)$$

The drag coefficient is given as a function of the particle Reynolds number:

$$Re_p = \frac{\rho_F D_p (\vec{u}_F - \vec{u}_p)}{\mu_F} \quad (3.7)$$

The dependence of the drag coefficient of a sphere (spherical particle) on the Reynolds number is shown in Figure 3.1 based on numerous experimental investigations (Schlichting 1965). From this dependence one may identify several regimes which are associated with the flow characteristics around the sphere:

- For small Reynolds numbers (i.e.  $Re_p < 0.5$ ) viscous effects are dominating and no separation is observed. Therefore, an analytic solution for the drag coefficient is possible as proposed by Stokes (1851):

$$C_D = \frac{24}{Re_p} \quad (3.8)$$

This regime is often referred to as the Stokes-regime.

- In the transition region (i.e.  $0.5 < Re_p < 1000$ ) inertial effects become of increasing importance. Above a Reynolds number of about 24 the flow around the particle begins to separate. Initially this separation is symmetric (Clift et al. 1978). It becomes unstable and periodic above  $Re_p \approx 130$ . For this non-linear regime numerous correlations have been proposed (Clift et al. 1978, Crowe et al. 1998) which fit the experimental data more or less accurate. A frequently used correlation is that proposed by Schiller & Naumann (1933), which fits the data up to  $Re_p = 1000$  reasonably well (see Figure 3.1).

$$c_D = \frac{24}{Re_p} \left( 1 + 0.15 Re_p^{0.687} \right) = \frac{24}{Re_p} f_D \quad (3.9)$$

- Above  $Re_p \approx 1000$  the drag coefficient remains almost constant up to the critical Reynolds number, since the wake size and structure is not considerably changing. This regime is referred to as Newton-regime with:

$$C_d \approx 0.44 \quad (3.10)$$

- At the critical Reynolds number ( $Re_{crit} \approx 2.5 \cdot 10^5$ ) a drastic decrease of the drag coefficient is observed, being caused by the transition from a laminar to a turbulent boundary layer around the particle. This results in a decrease of the particle wake.
- In the super-critical region (i.e.  $Re_p > 4.0 \cdot 10^5$ ) the drag coefficient again increases continuously. For most practical particulate flows however this region is not relevant.

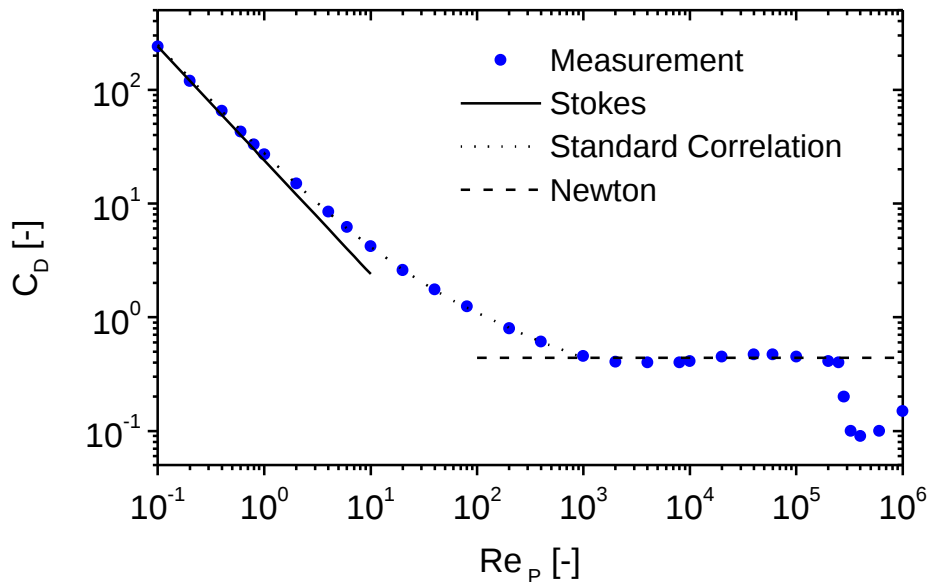


Figure 3.1: Drag coefficient as a function of particle Reynolds number, comparison of experimental data with the correlation for the different regimes (The standard correlation corresponds to Eq. 3.9)

The drag coefficient may be altered by numerous other physical effects, such as turbulence of the flow, surface roughness of the particle, particle shape, wall effects, compressibility of the fluid, rarefaction effects, which in general can be only accounted for by empirical correction factors or functions being derived from detailed experiments.

The **turbulence level of the ambient flow** essentially causes a reduction of the critical Reynolds number as shown by Torobin and Gauvin (1961). With increasing turbulence intensity the transition from laminar to turbulent boundary layer is shifted towards smaller particle Reynolds numbers.

A **surface roughness** on a spherical particle also results in a reduction of the critical Reynolds number (Sawatzki 1961).

The consideration of the **particle shape** in the calculation of particle motion is rather difficult, since it requires actually the solution of additional ordinary differential equations for the particle orientation and a projection of the forces with regard to the relative motion. Such an approach was recently introduced by Rosendahl (1998). Therefore, most computations rely on the assumption of spherical particles. A simplified approach to consider a non-sphericity of the particle may be based on the use of modified drag coefficients, which are provided for different non-spherical particles for example by Haider & Levenspiel (1989) and Thompson & Clark (1991). It is however very little known about particle shape effects in the other forces, such as added mass and transverse lift forces.

The **motion of particles in the vicinity of a rigid wall** results in an increase of the drag coefficient and is additionally associated with a transverse lift force. Analytic solutions for the wall effect are again only available for very small particle Reynolds numbers. The particle motion normal to a wall (Figure 3.2a)) was for example considered by Brenner (1961) and a wall-parallel motion (Figure 3.2b)) was analysed by Goldman et al. (1967). The first order solution for a particle moving towards a wall, which is valid for large distances from the wall, is given by (Brenner 1961):

$$\frac{C_D}{C_{D,Stokes}} \cong 1 + \frac{9}{8} \frac{R_p}{h} \quad (3.11)$$

For a non-rotating particle moving parallel to a wall in a quiescent fluid the increase of the drag is predicted by an asymptotic solution proposed by Faxen (1923) for large distances from the wall:

$$\frac{C_D}{C_{D,Stokes}} = \left[ 1 - \frac{9}{16} \left( \frac{R_p}{h} \right) + \frac{1}{8} \left( \frac{R_p}{h} \right)^3 - \frac{45}{256} \left( \frac{R_p}{h} \right)^4 - \frac{1}{16} \left( \frac{R_p}{h} \right)^5 \right]^{-1} \quad (3.12)$$

The two results are shown in Figure 3.3 as a function of the normalised gap between particle and wall (i.e.  $a/R_p$ ). For large wall distance the curves approach unity and a finite value is obtained for  $a/R_p \rightarrow 0$ . It should be noted that wall effects will be additionally affected by particle rotation and a shear flow in the vicinity of the wall Goldman et al. (1967).

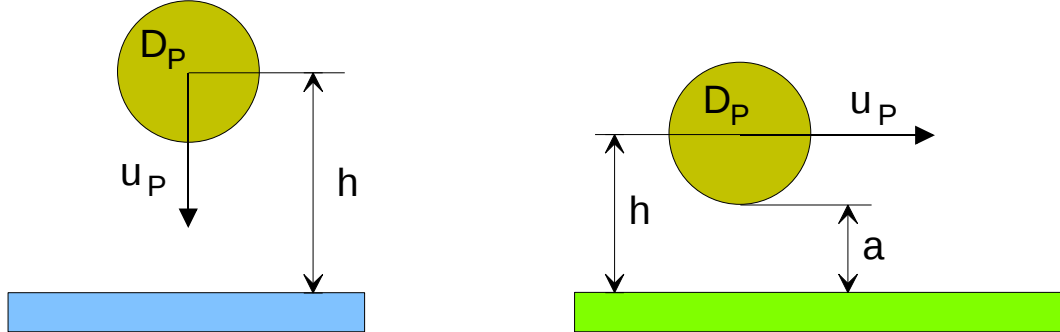


Figure 3.2: Illustration of wall effects, a) motion normal to a wall, b) motion parallel to a wall

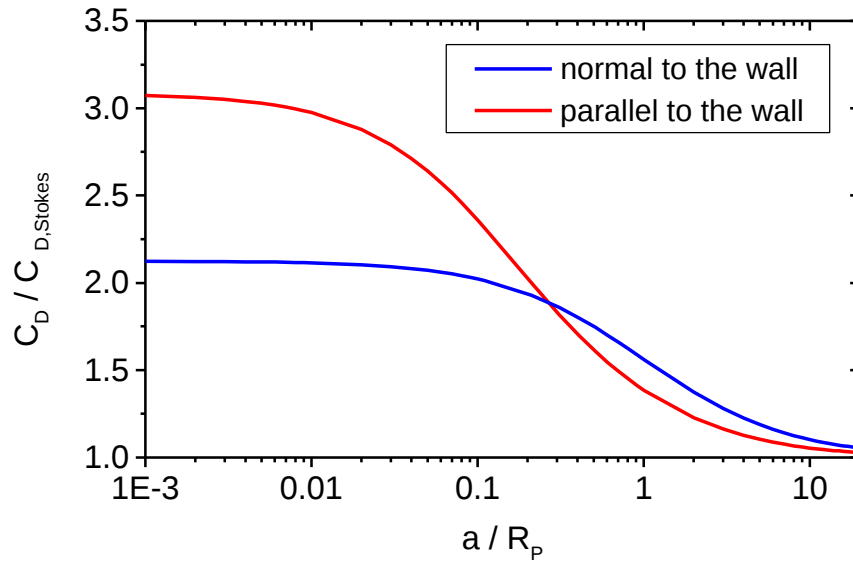


Figure 3.3: Modification of drag coefficient for a particle moving normal and parallel to a wall (Eqs. 3.11 and 3.12)

**Rarefaction effects** become of importance in a low pressure environment or when the particles are very small. In such a situation the gas flow around the particle cannot be regarded as a continuum, instead the particle motion is induced by collisions of gas molecules with the particle surface. This results in a reduction of the drag coefficient. The importance of rarefaction effects may be estimated based on the ratio of the mean free path of the gas molecules to the particle diameter, which is the particle Knudsen number:

$$Kn_p = \frac{\lambda}{D_p} \quad (3.13)$$

The mean free path of the gas molecules  $\lambda$  can be calculated according to kinetic theory of gases from:

$$\lambda = \frac{\mu_F}{0.499 \bar{c}_{Mol} \rho_F} \quad (3.14)$$

where  $\bar{c}_{Mol}$  is the mean relative velocity between gas molecules given by:

$$\bar{c}_{Mol} = \left( \frac{8p}{\pi \rho_F} \right)^{1/2} \quad (3.15)$$

and  $p$  is the pressure. For atmospheric conditions (i.e.  $p = 1.0$  bar,  $T = 293$  K) the mean free path is about  $0.06 \mu m$ . A classification of the different flow regimes in rarefied conditions or for very small particles may be based on the Knudsen number and is summarised in Table 3.1. In the Stokes regime which is generally valid for very small particles, the reduction of the drag coefficient may be accounted for by a correction function, the so-called Cunningham correction, resulting in a modified drag (Davies 1945):

$$C_D = \frac{C_{D,Stokes}}{1 + Kn_p \left\{ 2.514 + 0.8 \exp\left(-\frac{0.55}{Kn_p}\right) \right\}} = \frac{C_{D,Stokes}}{Cu} \quad (3.16)$$

This correlation is valid for  $0.1 < Kn_p < 1000$  and  $Re_p < 0.25$  and is only applicable for low particle Mach numbers (definition see below). Therefore, it is often used in particle technology, as for example when considering the separation of fine particles from a gas. The Cunningham correction, i.e.  $1/Cu$ , is plotted in Figure 3.4 as a function of the Knudsen number. It is obvious, that a considerable reduction of the drag coefficient occurs for  $Kn_p > 0.015$ .

| Flow Regime         | Range of Knudsen number |
|---------------------|-------------------------|
| Continuum flow      | $0 < Kn_p < 0.015$      |
| Slip flow           | $0.015 < Kn_p < 0.15$   |
| Transition flow     | $0.15 < Kn_p < 4.5$     |
| Free molecular flow | $4.5 < Kn_p < \infty$   |

Table 3.1 Different regimes of rarefied flows with respect to particle motion

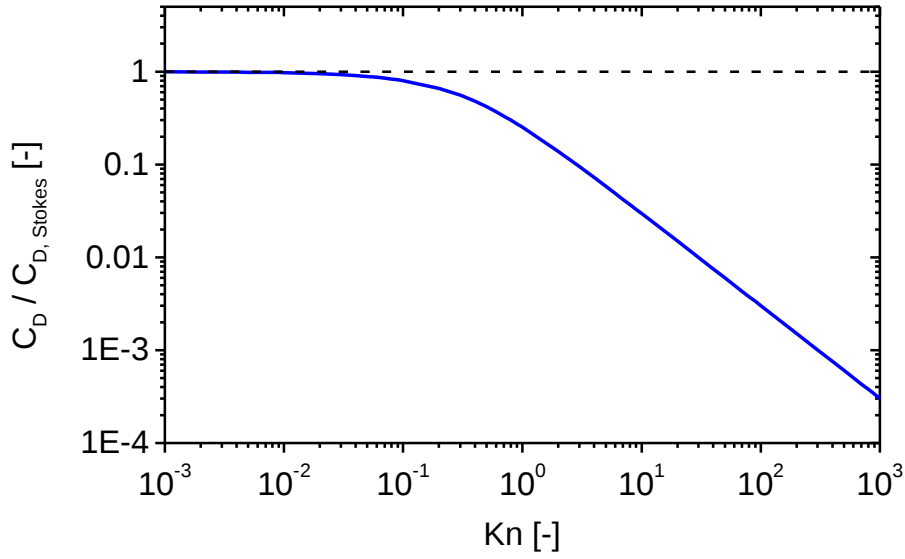


Figure 3.4: Modification of the drag coefficient due to rarefaction effects (Eq. 3.16)

The compressibility of the fluid (i.e. a gas) becomes of importance when the relative velocity becomes so large that the particle Mach number increases beyond 0.3. In such a situation compression waves or even shock waves (for  $Ma_p > 1$ ) are initiated by the particle motion which cause an increase of the drag for large particle Reynolds numbers. The particle Mach number is defined as:

$$Ma_p = \frac{|\vec{u}_F - \vec{u}_p|}{a} \quad (3.17)$$

where  $a$  is the speed of sound given by:

$$a = \sqrt{\gamma R T} \quad (3.18)$$

In Eq. (3.18),  $\gamma$  is the ratio of the specific heats,  $R$  is the universal gas constant, and  $T$  the absolute temperature of the gas.

Numerous correlations, which are mostly based on experimental studies, are proposed to account for compressibility effects, as for example the following expression proposed by Carlson and Hoglund (1964):

$$C_D = C_{D,0} \frac{1.0 + \exp\left(-\frac{0.427}{Ma_p^{4.63}} - \frac{3.0}{Re_p^{0.88}}\right)}{1.0 + \frac{Ma_p}{Re_p} \left\{ 3.82 + 1.28 \exp\left(-1.25 \frac{Re_p}{Ma_p}\right) \right\}} \quad (3.19)$$

where  $C_{D,0}$  is the drag coefficient given by Eqs. 3.9 and 3.10. The term in the nominator accounts for compressibility, while the denominator accounts for rarefaction effects. The drag

coefficient versus Mach number is plotted in Figure 3.5 for a small and large particle Reynolds number. For small particles the drag coefficient is decreasing due to rarefaction effects, whereas it increases for large particles beyond a Mach number of about 0.6 due to compressibility effects. Other expressions for the drag coefficient including rarefaction and compressibility effects are given by Crowe et al. (1998).

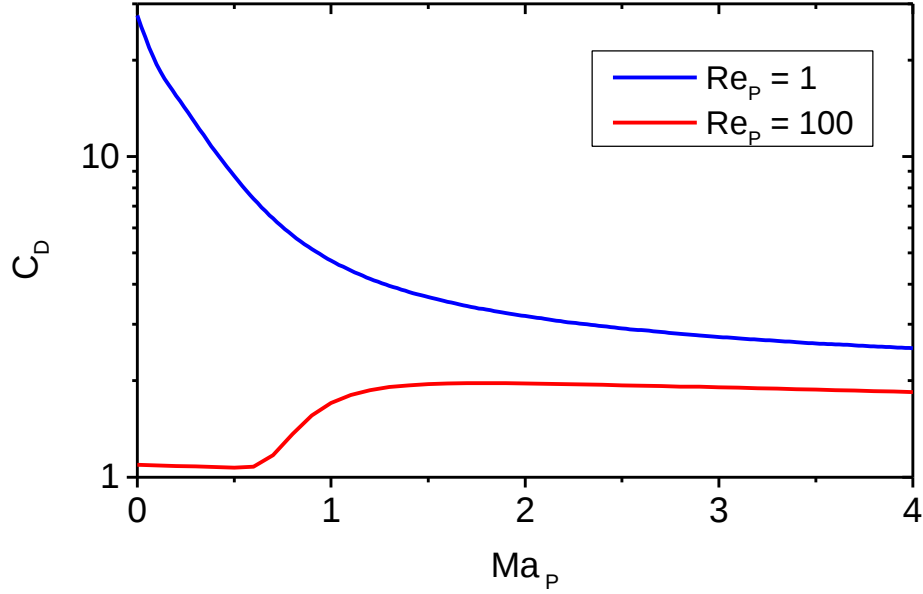


Figure 3.5: Modification of the drag coefficient due to rarefaction and compressibility effects for small and large particles (Eq. 3.19)

### 3.2 Pressure Gradient and Buoyancy Force

The local pressure gradient in the flow gives rise to an additional force in the direction of the pressure gradient. Combining the pressure gradient with the shear stress in the fluid, one obtains:

$$\mathbf{F}_p = \frac{m_p}{\rho_p} (-\nabla p + \nabla \vec{\tau}) \quad (3.20)$$

From the Navier-Stokes equation of the fluid the pressure gradient and the shear stress can be related to the fluid acceleration and the gravity force:

$$-\nabla p + \nabla \vec{\tau} = \rho_F \left( \frac{D\vec{u}_F}{Dt} - \vec{g} \right) \quad (3.21)$$

Hence the total pressure force is obtained in the following form:

$$\vec{F}_p = m_p \frac{\rho_F}{\rho_p} \left( \frac{D\vec{u}_F}{Dt} - \vec{g} \right) \quad (3.22)$$

The first term of Eq. 3.22 represents the fluid acceleration and the second one is the buoyancy force. It is obvious, that in gas solid flows the pressure force may be neglected since  $\rho_F/\rho_p \ll 1$ . However, in liquid solid flows this force is of importance since  $\rho_F/\rho_p \approx 1$ .

### 3.3 Added Mass and Basset Force

The acceleration/deceleration of a particle in a fluid also requires to accelerate/decelerate a certain fraction of the surrounding fluid, this is the so-called added mass. The Basset force is caused by the lagging of the boundary layer development on the particle with changing relative velocity (i.e. acceleration or deceleration of the particle) and is often referred to as „history“ force. Analytic solutions for both forces are only possible for small particle Reynolds numbers (see Eq. 3.1). An extension to higher particle Reynolds numbers is only possible by introducing empirical coefficients similar to the drag coefficient for all the forces. Based on an experimental study of Odar and Hamilton (1964) who studied the motion of a sphere in simple harmonic motion, the added mass and Basset force are expressed as:

$$F_A = 0.5 C_A \rho_F \frac{m_p}{\rho_p} \frac{d}{dt} (\bar{u}_F - \bar{u}_p) \quad (3.23)$$

$$F_B = 9 \sqrt{\frac{\rho_F \mu_F}{\pi}} \frac{m_p}{\rho_p D_p} C_B \left\{ \int_0^t \frac{d}{dt} (\bar{u}_F - \bar{u}_p) \frac{d\tau}{(t-\tau)^{1/2}} + \frac{(\bar{u}_F - \bar{u}_p)_0}{\sqrt{t}} \right\} \quad (3.24)$$

The second term in the Basset force accounts for an initial slip velocity at  $t = 0$  (Reeks and McKee 1984). The coefficients  $C_A$  and  $C_B$  were obtained from the experiments of Odar and Hamilton (1964) in the following form:

$$C_A = 2.1 - \frac{0.132}{A_C^2 + 0.12} \quad (3.25)$$

$$C_B = 0.48 + \frac{0.52}{(A_C + 1)^3} \quad (3.26)$$

The parameter  $A_C$  is called acceleration number and is defined by:

$$A_C = \frac{|\bar{u}_F - \bar{u}_p|^2}{D_p \left| \frac{d|\bar{u}_F - \bar{u}_p|}{dt} \right|} \quad (3.27)$$

It is obvious that the Basset force is quite time consuming to solve since it has to be integrated along the entire particle trajectory. Therefore, this force is often neglected. An approximate solution procedure for the Basset force was introduced by Michaelides (1992). Numerical

calculations of Sommerfeld (1996) have shown that the consideration of the Basset force increases the computational time by a factor of about 10. An analysis of the importance of the different forces, especially added mass and Basset force, in an oscillatory flow field for different density ratios will be provided below.

### 3.4 Body Forces

Body forces are the gravity force, the Coulomb force, which arises when a particle moves in an electric field, as for example in an electrostatic precipitator or the thermophoretic force which becomes of importance when a small particle moves in a flow with a high temperature gradient.

The gravity force is:

$$\vec{F}_g = m_p \vec{g} . \quad (3.28)$$

The Coulomb force acting on a particle moving in an electric field with a field intensity  $\vec{E}$  is given by:

$$F_C = -q_p \vec{E} \quad (3.29)$$

where  $q_p$  is the charge of the particle. In an electrostatic precipitator, for example, the particles are charged by an ion-bombardment created by a negative corona discharge in the vicinity of a charging wire. The charging of the particles is caused by two mechanisms: field charging and/or diffusion charging.

**Field charging** occurs due to the convective motion of the ions and is relevant for particles larger than about 0.5 to 1  $\mu\text{m}$ . The saturation charge for a spherical particle is:

$$q_p = \pi \epsilon_0 D_p^2 E_0 p \quad (3.30)$$

where  $\epsilon_0$  is the permittivity of the free space,  $E_0$  is the electric field strength in the charging region and  $p = 3$  for conducting particles and  $p = 1.5$  to  $2.0$  for non-conducting particles.

**Diffusion charging** is the result of the thermal motion of the ions and is relevant for particles with a diameter smaller than about 0.2  $\mu\text{m}$ . The rate of charge increase due to diffusion charging is given by:

$$q_p(t) = 4 \pi \epsilon_0 \frac{k \cdot T}{e} \frac{x}{2} \ln \left\{ 1 + \frac{x c N_0 e^2 t}{8 \epsilon_0 k T} \right\} \quad (3.31)$$

where  $k$  is the Boltzmann constant,  $e$  is the elementary charge,  $N_0$  is the number density of the ions,  $c$  is the mean fluctuating velocity of the ions,  $t$  is the time, and  $T$  the absolute temperature of the gas. More details about particle charging and their motion in an electric field may be found in the book of White (1963).

### 3.5 Slip-Shear Lift Force

Particles moving in a shear layer experience a transverse lift force due to the non-uniform relative velocity over the particle and the resulting non-uniform pressure distribution. The lift force is acting towards the direction of higher slip velocity (Figure 3.6). An expression for the slip shear lift force for a freely rotating particle moving at constant velocity in a two-dimensional shear flow at low Reynolds number was derived from an asymptotic expansion by Saffman (1965, 1968):

$$F_{LS,Saff}^y = 6.46 \frac{D_p^2}{4} (\rho_F \mu_F)^{0.5} \left| \frac{\partial u_F}{\partial y} \right| (u_F - u_p) \quad (3.32)$$

Extending this expression to a three dimensional flow and introducing a correction function for higher particle Reynolds numbers yields:

$$\vec{F}_{LS} = 1.615 D_p^2 (\rho_F \mu_F)^{1/2} \left( \frac{1}{|\vec{\omega}_F|} \right)^{0.5} \{ (\vec{u}_F - \vec{u}_p) \times \vec{\omega}_F \} f(Re_p, Re_s) \quad (3.34)$$

Here the fluid rotation is obtained from:

$$\vec{\omega}_F = \text{rot } \vec{u}_F = \nabla \times \vec{u}_F \quad (3.35)$$

Introducing now a lift coefficient in Eq. (3.34) gives the following impression for the slip-shear lift force:

$$\vec{F}_{LS} = \frac{\rho_F}{2} \frac{\pi}{4} D_p^2 C_{LS} D_p ((\vec{u}_F - \vec{u}_p) \times \vec{\omega}_F) \quad (3.36)$$

with the lift coefficient:

$$C_{LS} = \frac{4.1126}{Re_s^{0.5}} f(Re_p, Re_s) \quad (3.37)$$

The correction function  $f(Re_p, Re_s)$  proposed by Mei (1992) on the basis of calculations performed by Dandy and Dwyer (1990) for a particle Reynolds number in the range  $0.1 \leq Re_p \leq 100$  is given by:

$$f(Re_p, Re_s) = \frac{F_{LS}}{F_{LS,Saff}} \quad (3.38)$$

$$\begin{aligned} \frac{F_{LS}}{F_{LS,Saff}} &= \left( 1 - 0.3314 \beta^{1/2} \right) \exp\left( -\frac{Re_p}{10} \right) + 0.3314 \beta^{1/2} \quad \text{for : } Re_p \leq 40 \\ &= 0.0524 (\beta Re_p)^{1/2} \quad \text{for : } Re_p \geq 40 \end{aligned} \quad (3.39)$$

with:

$$\beta = 0.5 \frac{Re_s}{Re_p} \quad (3.40)$$

and the Reynolds number of the shear flow:

$$\text{Re}_s = \frac{\rho_F D_p^2 |\vec{\omega}_F|}{\mu_F} \quad (3.41)$$

The dependence of the lift coefficient on the particle Reynolds number with the shear Reynolds number as a parameter is shown in Figure 3.7. The horizontal lines indicate the values for the Saffman expression which agree with the lift coefficient in Eq. 3.37 only for small Reynolds numbers.

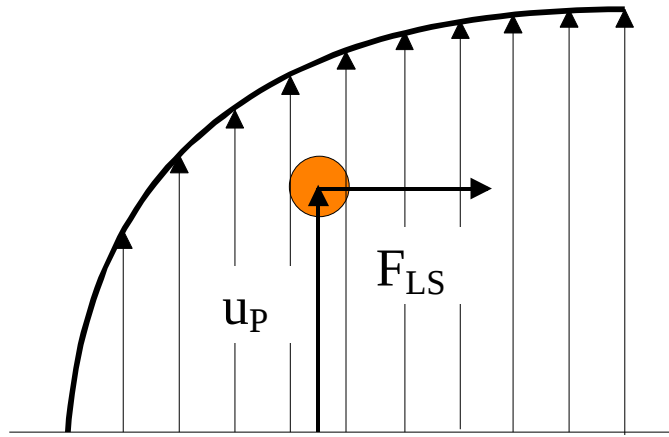


Figure 3.6: Illustration of the slip-shear lift force

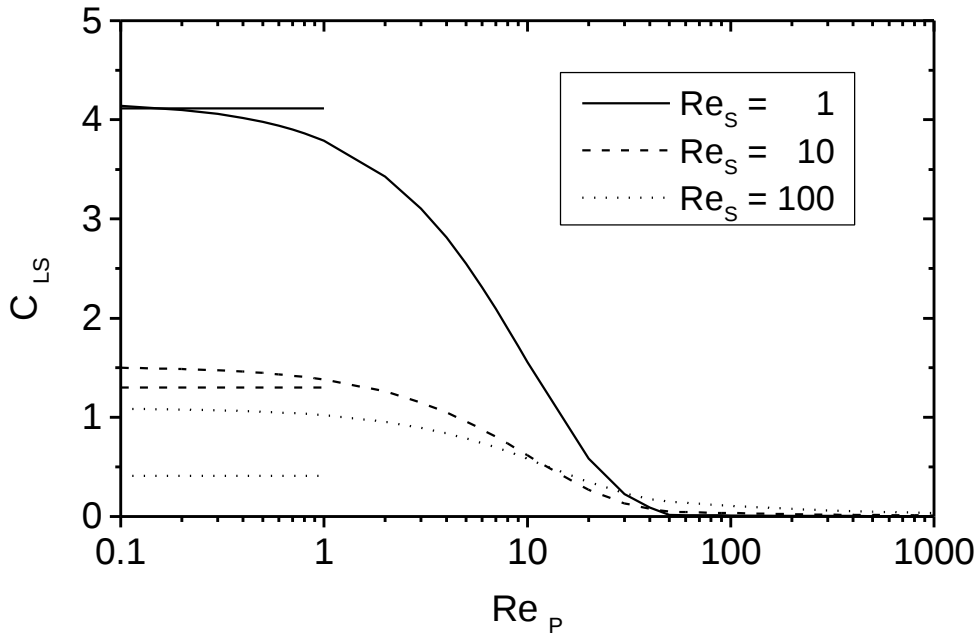


Figure 3.7: Lift coefficient as a function of particle Reynolds number with the shear Reynolds number as a parameter (The horizontal line indicate the lift coefficient of the Saffman lift force, Eq. 3.37 with  $f(\text{Re}_p, \text{Re}_s) = 1.0$ ).

### 3.6 Slip-Rotation Lift Force

Particles which are not freely rotating in a flow may also experience a lift force due to their rotation, the so-called Magnus force. High particle rotations may for example be induced by particle-wall collision frequently occurring in pipe or channel flows. The rotation of the particle results in a deformation of the flow field around the particle, associated with a shift of the stagnation points and a transverse lift force (Figure 3.8). An analytic expression for the slip-rotation lift force in the case of small particle Reynolds numbers was derived by Rubinow and Keller (1961):

$$\vec{F}_{LR} = \pi R_p^3 \rho_F \left\{ \vec{\Omega} \times (\vec{u}_F - \vec{u}_p) \right\} \quad (3.42)$$

where  $\vec{\Omega}$  is the relative rotation given by:

$$\vec{\Omega} = \frac{1}{2} \nabla \times \vec{u}_F - \vec{\omega}_p \quad (3.43)$$

Also the slip-rotation lift force may be extended for higher particle Reynolds numbers by introducing a lift coefficient (Crowe et al. 1998):

$$\vec{F}_{LR} = \frac{\rho_F \pi}{2} \frac{D_p^2}{4} C_{LR} |\vec{u}_F - \vec{u}_p| \frac{\vec{\Omega} \times (\vec{u}_F - \vec{u}_p)}{|\vec{\Omega}|} \quad (3.44)$$

For small particle Reynolds numbers the lift coefficient is obtained according to Rubinow and Keller (1961) in the form:

$$C_{LR} = \frac{D_p |\vec{\Omega}|}{|\vec{u}_F - \vec{u}_p|} = \frac{Re_R}{Re_p} \quad (3.45)$$

with:

$$Re_R = \frac{\rho_F D_p^2 |\vec{\Omega}|}{\mu_F} \quad (3.46)$$

being the Reynolds number of particle rotation. A lift coefficient for higher particle Reynolds numbers requires experimental information. Recently, Oesterlé and Bui Dinh (1998) introduced the following correlation based on available literature data and additional experiments for  $Re_p < 140$ :

$$C_{LR} = 0.45 + \left( \frac{Re_R}{Re_p} - 0.45 \right) \exp \left( -0.05684 \cdot Re_R^{0.4} \cdot Re_p^{0.3} \right) \quad \text{for : } Re_p < 140 \quad (3.47)$$

The lift coefficient of particle rotation as a function of the particle Reynolds number with the Reynolds number of particle rotation as a parameter is shown in Figure 3.9. The upper and

lower straight lines correspond to the result of Rubinow and Keller (1961) given by Eq. 3.45. It is obvious that this expression only holds for small particle Reynolds numbers.

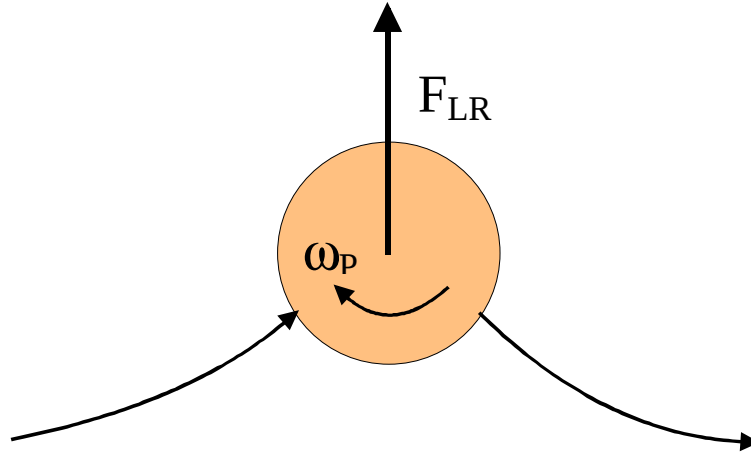


Figure 3.8: Illustration of the slip-rotation lift force acting on a stationary particle

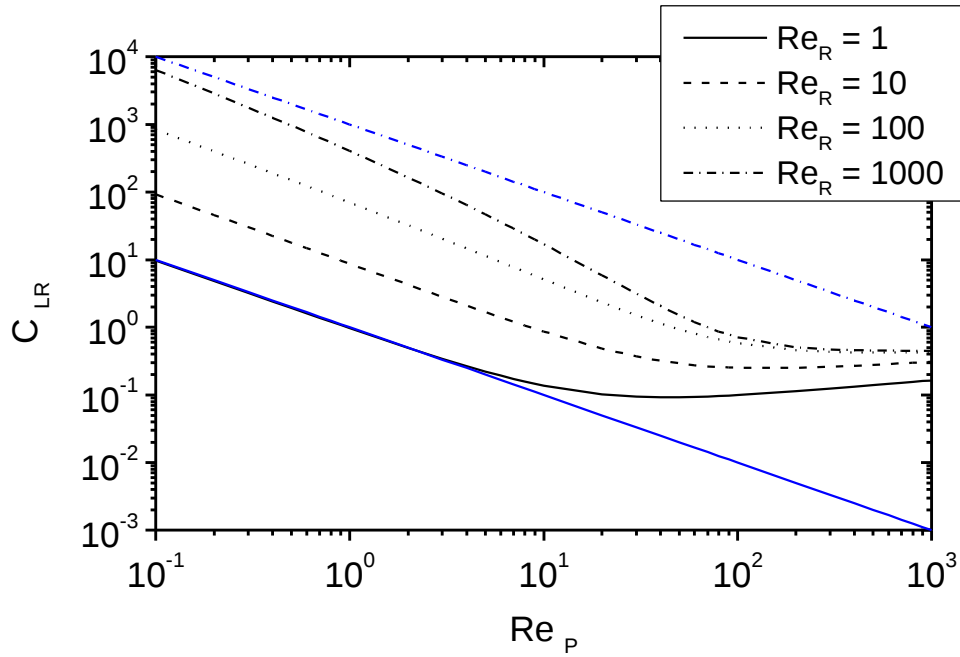


Figure 3.9: Lift coefficient of particle rotation as a function of particle Reynolds number with the Reynolds number of particle rotation as a parameter. The lower and upper straight lines correspond to the result of Rubinow and Keller (1961) for  $Re_R = 1$  and  $Re_R = 1000$ , respectively.

### 3.7 Torque

The torque acting on a rotating particle due to the interaction with the fluid was also derived by Rubinow and Keller (1961) for a stagnant fluid and small particle Reynolds numbers:

$$\vec{T} = -\pi \mu_F D_p^3 \vec{\omega}_p \quad (3.48)$$

This expression may be extended for a three-dimensional flow and for higher Reynolds numbers by introducing a rotational coefficient:

$$\vec{T} = \frac{\rho_F}{2} \left( \frac{D_p}{2} \right)^5 C_R |\vec{\Omega}| \vec{\Omega} \quad (3.49)$$

From the numerical simulations of Dennis et al. (1980) and experimental data of Sawatzki (1970) the rotational coefficient for higher particle Reynolds numbers is found to be:

$$C_R = \frac{12.9}{Re_R^{0.5}} + \frac{128.4}{Re_R} \quad \text{for : } 32 < Re_R < 1000 \quad (3.50)$$

In the case of smaller particle Reynolds numbers the result of Rubinow and Keller (1961) yields:

$$C_R = \frac{64 \pi}{Re_R} \quad \text{for : } Re_R < 32 \quad (3.51)$$

The comparison of the above correlation (Eqs. 3.50 and 3.51) with the simulations (Dennis et al. 1980) and the experiments (Sawatzki 1970) give a good agreement as shown in Figure 3.10.

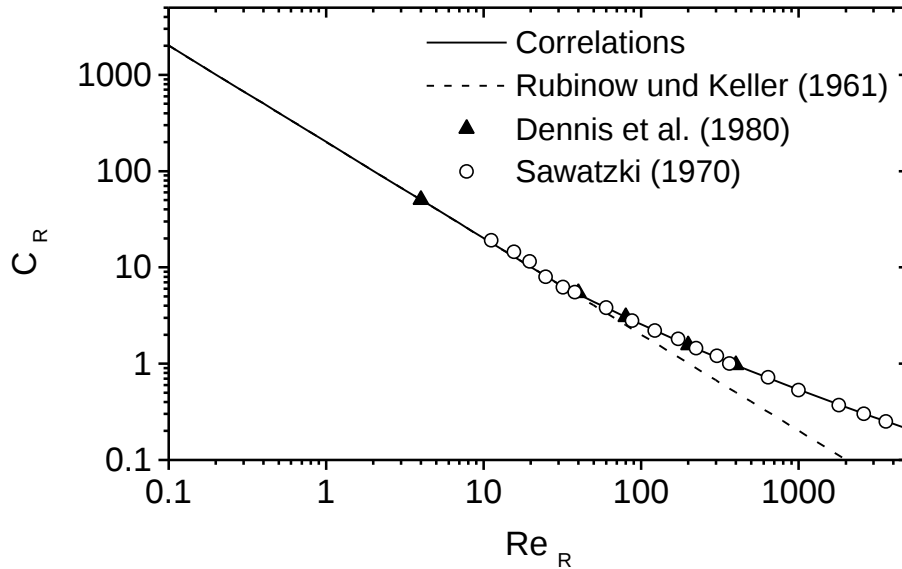


Figure 3.10: Coefficient of particle rotation as a function of particle rotational Reynolds number according to Eqs. 3.50 and 3.51 and comparison with experiments (Sawatzki 1970) and numerical calculations (Dennis et al. 1980)

### 3.8 Response Time and Stokes Number

The particle (velocity or momentum) response time may be used to characterise the capability of particles to follow a sudden velocity change in the flow, occurring for example in large scale vertical structures or turbulent eddies. In order to derive the particle response time the equation of motion is used by only considering the drag force.

$$m_p \frac{du_p}{dt} = \frac{\rho_F}{2} \frac{\pi}{4} D_p^2 C_D |u_F - u_p| (u_F - u_p) \quad (3.52)$$

Dividing by the particle mass and introducing the particle Reynolds number gives:

$$\frac{du_p}{dt} = \frac{18 \mu_F}{\rho_p D_p^2} \frac{C_D Re_p}{24} (u_F - u_p) \quad (3.53)$$

The term  $C_D Re_p/24$  corresponds to the non-linear term in the drag coefficient  $f_D$  and the first term of Eq. 3.53 has the dimension of a time, the particle response time:

$$\tau_p = \frac{\rho_p D_p^2}{18 \mu_F f_D} \quad (3.54)$$

Hence the equation of motion becomes:

$$\frac{du_p}{dt} = \frac{1}{\tau_p} (u_F - u_p) \quad (3.55)$$

The solution of this equation for a simplified case, namely a constant fluid velocity  $u_F$  and an initial particle velocity of zero is:

$$u_p = u_F \left( 1 - \exp\left(-\frac{t}{\tau_p}\right) \right) \quad (3.56)$$

From this equation it is obvious that  $\tau_p$  is the time required for a particle released with zero velocity into a flow with  $u_F$  to reach 63.2 % of the flow velocity all illustrated in Figure 3.11.

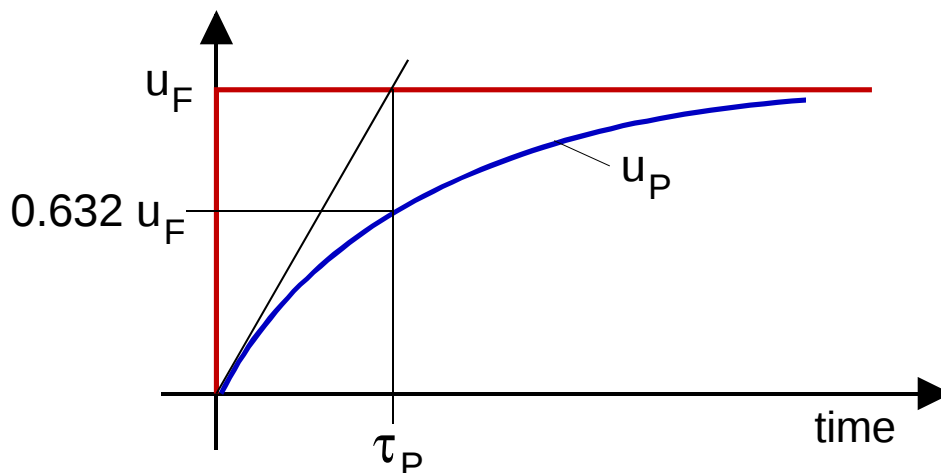


Figure 3.11: Graphical illustration of the particle response time

In the Stokes-regime the particle response time becomes:

$$\tau_p = \frac{\rho_p D_p^2}{18 \mu} \quad (3.57)$$

since  $f_D$  approaches unity.

The Stokes number is the ratio of the particle response time to a characteristic time scale of the flow.

$$St = \frac{\tau_p}{\tau_F} \quad (3.58)$$

### 3.9 Importance of the Different Forces

In order to estimate the importance of the different forces, especially the importance of added mass and Basset force, acting on a particle in a turbulent flow, Hjelmfeld & Mockros (1966) have performed an analysis for an oscillatory flow field. The starting point of their analysis was the Stokes form of the equation of motion given by:

$$\begin{aligned} m_p \frac{du_p}{dt} = & \frac{18 \mu_F}{\rho_p D_p^2} m_p (u_F - u_p) - m_F \frac{du_F}{dt} + 0.5 m_F \left( \frac{du_F}{dt} - \frac{du_p}{dt} \right) \\ & + 9 \sqrt{\frac{\rho_F \mu_F}{\pi}} \frac{m_p}{\rho_p D_p} \int_{-\infty}^t \frac{\frac{du_F}{d\tau} - \frac{du_p}{d\tau}}{(t - \tau)^{1/2}} d\tau \end{aligned} \quad (3.59)$$

Rearranging this equation results in:

$$\frac{du_p}{dt} + a u_p + c \int_{-\infty}^t \frac{du_p/d\tau}{(t - \tau)^{1/2}} d\tau = a u_F + b \frac{du_F}{dt} + c \int_{-\infty}^t \frac{du_F/d\tau}{(t - \tau)^{1/2}} d\tau \quad (3.60)$$

with the coefficients  $a$ ,  $b$  and  $c$  defined by:

$$a = \frac{18 \mu_F / \rho_F}{(\rho_p / \rho_F + 0.5) D_p^2}, \quad b = \frac{3}{2 (\rho_p / \rho_F + 0.5)}, \quad c = \frac{9}{(\rho_p / \rho_F + 0.5)} \sqrt{\frac{\mu_F}{\pi \rho_F}} \quad (3.61)$$

The velocities of the particles and the fluid are expressed by Fourier integrals:

$$u_F = \int_0^\infty (\zeta \cos \omega t + \lambda \sin \omega t) d\omega, \quad u_p = \int_0^\infty (\sigma \cos \omega t + \phi \sin \omega t) d\omega \quad (3.62)$$

where  $\omega$  is the frequency of oscillation. Introducing these Fourier integrals into the equation of motion of the particles (Eq. 3.60) yields the amplitude ratio (i.e. amplitude of particle velocity over that of the fluid) and the phase angle (i.e. lag of particle response) in the following form:

$$\eta = \sqrt{(1 + f_1)^2 + f_2^2}, \quad \beta = \tan^{-1} \left\{ \frac{f_2}{1 + f_1} \right\} \quad (3.63)$$

The functions  $f_1$  and  $f_2$  are obtained as:

$$f_1 = \frac{\omega (\omega + c \sqrt{0.5 \pi \omega}) (b - 1)}{(a + c \sqrt{0.5 \pi \omega})^2 + (\omega + c \sqrt{0.5 \pi \omega})^2} \quad (3.64)$$

$$f_2 = \frac{\omega (a + c \sqrt{0.5 \pi \omega}) (b - 1)}{(a + c \sqrt{0.5 \pi \omega})^2 + (\omega + c \sqrt{0.5 \pi \omega})^2} \quad (3.65)$$

The parameter used to characterise the particle response is a modified Stokes number given by:

$$N_s = \frac{\mu_F}{\rho_F \omega D_p^2} \quad (3.66)$$

The result of this analysis is shown in Figure 3.12 for three kinds of particles, namely copper and glass particles in air and air bubbles in water, by considering the different forces. For the three cases the amplitude ratio and the phase angle is plotted versus the modified Stokes number. It is obvious that for copper particles and glass beads the added mass, the pressure force, and the Basset term have almost no effect on the amplitude ratio. However, considerable differences are observed in the phase angle for  $N_s < 5$ , which means for large particles or high frequencies of the oscillatory fluid motion. Only the added mass is not of great importance and may be neglected without considerable error. Considering a 100  $\mu\text{m}$  particle the pressure force and the Basset term become of importance for oscillation frequencies larger than about 310 Hz. For bubbly flows large differences in the response arise for  $N_s < 1.0$ . However, for this case the Basset term may be neglected without introducing very large errors. The added mass and the pressure force on the other hand are of great importance.

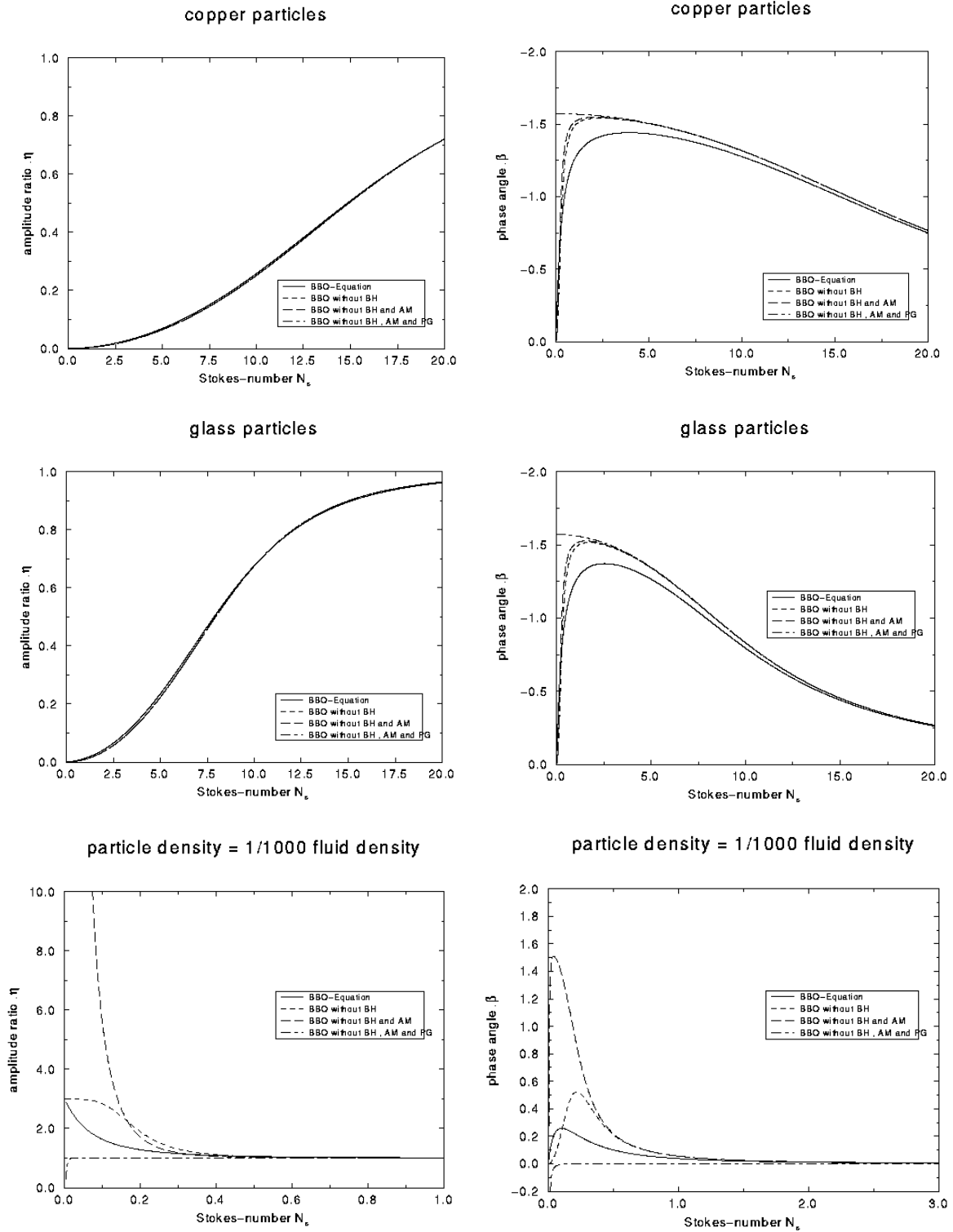


Figure 3.12: Particle response in an oscillatory flow field, influence of the different forces on the amplitude ratio (left column) and the phase angle (right column) for: a) copper particles in air, b) glass beads in air, c) air bubbles in water

## 4. Particle-Wall Collisions

Particle-wall collisions become of importance in confined flows, such as pneumatic conveying or particle separation in cyclones. In pneumatic conveying, for example, the momentum loss of a particle caused by an inelastic wall impact is associated with a re-acceleration of the particle after rebound. Hence, momentum is extracted from the fluid phase for this acceleration, causing an additional pressure loss. This pressure loss depends on the average wall collision frequency or mean free path between subsequent particle-wall collisions. The wall collision frequency is mainly determined by the following parameters:

- particle mass loading
- dimensions of the confinement, e.g. pipe diameter in pneumatic conveying
- particle response time or response distance
- conveying velocity and turbulence intensity
- particle shape and wall roughness
- combination of particle and wall material.

A first estimate of the importance of particle-wall collisions may be based on the ratio of the particle response distance  $\lambda_p$  to the dimension of the confinement, e.g. the diameter of the pipe  $D$ . The particle response distance can be estimated from the following equation:

$$\lambda_p = \frac{\rho_p D_p^2}{18 \mu_F f_D} \cdot w_t \quad (4.1)$$

where  $w_t$  is the terminal velocity of the particle. For the case  $\lambda_p$  is larger than the dimension of the confinement  $D$ , the particles are not able to respond to the flow, before they collide with the opposite wall, hence their motion is dominated by wall collisions. In addition to the above mentioned effects the wall collision process may be affected by hydrodynamic interaction which eventually causes a deceleration of the particle before impact (see above, the section on wall effects). This effect however is only of importance for viscous fluids and hence small particle Reynolds numbers.

### 4.1 Velocity Change During Wall Collisions

In the following the so-called hard sphere model for the wall collision will be described which implies a negligible particle deformation during the impact process. Moreover, Coulomb's law of friction is assumed to hold for a sliding collision. For an inelastic collision process, one may identify a compression and a recovery period. The change of the particles translational and rotational velocities during the bouncing process can be calculated from the momentum

equations of classical mechanics (Crowe et al. 1998). Three types of collisions may be distinguished:

Type 1: The particle stops sliding in the compression period.

Type 2: The particle stops sliding in the recovery period.

Type 3: The particle continues to slide along the wall during the whole collision process.

The type of collision is determined by the static coefficient of friction  $\mu_0$ , the restitution ratio of the normal velocity components,  $e$ , and the velocity of the particle surface relative to the contact point,  $u_{R1}$ . The **non-sliding collision** (type 1 and 2) takes place when the following condition is valid:

$$|u_{R1}| \leq \frac{7}{2} \mu_0 (1+e) v_{p1} \quad (4.2)$$

$$u_{R1} = \sqrt{\left(u_{p1} + \frac{D_p}{2} \omega_{p1}^z\right)^2 + \left(w_{p1} - \frac{D_p}{2} \omega_{p1}^x\right)^2} \quad (4.3)$$

where,  $u_p$ ,  $v_p$ , and  $w_p$  are the translational velocity components and  $\omega_p^x$ ,  $\omega_p^y$ , and  $\omega_p^z$  are the angular velocity components of the particle in a co-ordinate system as shown in Figure 3.12. The subscripts 1 and 2 refer to the conditions before and after collision, respectively. For the non-sliding collision, the change of particle velocities is obtained by:

$$\begin{aligned} u_{p2} &= \frac{5}{7} \left( u_{p1} - \frac{D_p}{5} \omega_{p1}^z \right) \\ v_{p2} &= -e v_{p1} \\ w_{p2} &= \frac{5}{7} \left( w_{p1} + \frac{D_p}{5} \omega_{p1}^x \right) \end{aligned} \quad (4.4)$$

$$\begin{aligned} \omega_{p2}^x &= \frac{2 w_{p2}}{D_p} \\ \omega_{p2}^y &= \omega_{p1}^y \\ \omega_{p2}^z &= -\frac{2 u_{p2}}{D_p} \end{aligned} \quad (4.5)$$

The collision type 3 is the so-called **sliding collision** which occurs for:

$$|u_{R1}| \geq \frac{7}{2} \mu_0 (1+e) v_{p1} \quad (4.6)$$

The change of translational and rotational velocities throughout the sliding collision is obtained by:

$$\begin{aligned}
\mathbf{u}_{p2} &= \mathbf{u}_{p1} + \mu_d \boldsymbol{\varepsilon}_x (1+e) v_{p1} \\
\mathbf{v}_{p2} &= -e v_{p1} \\
\mathbf{w}_{p2} &= \mathbf{w}_{p1} + \mu_d \boldsymbol{\varepsilon}_z (1+e) v_{p1}
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
\omega_{p2}^x &= \omega_{p1}^x - 5 \mu_d \boldsymbol{\varepsilon}_z (1+e) \frac{v_{p1}}{D_p} \\
\omega_{p2}^y &= \omega_{p1}^y \\
\omega_{p2}^z &= \omega_{p1}^z + 5 \mu_d \boldsymbol{\varepsilon}_x (1+e) \frac{v_{p1}}{D_p}
\end{aligned} \tag{4.8}$$

In Eqs. 4.7 and 4.8 the terms  $\boldsymbol{\varepsilon}_x$  and  $\boldsymbol{\varepsilon}_z$  determine the direction of the motion of the particle surface with respect to the wall:

$$\begin{aligned}
\boldsymbol{\varepsilon}_x &= \frac{\mathbf{u}_{p1} + \frac{D_p}{2} \boldsymbol{\omega}_{p1}^z}{u_{R1}} \\
\boldsymbol{\varepsilon}_z &= \frac{\mathbf{w}_{p1} - \frac{D_p}{2} \boldsymbol{\omega}_{p1}^x}{u_{R1}}
\end{aligned} \tag{4.9}$$

In the above equations  $e$  is the restitution ratio,  $\mu_0$  and  $\mu_d$  are the static and dynamic coefficients of friction. Unfortunately, these parameters are not only dependent on the material of particle and wall, but also on impact velocity and angle (see for example Sommerfeld & Huber 1999).

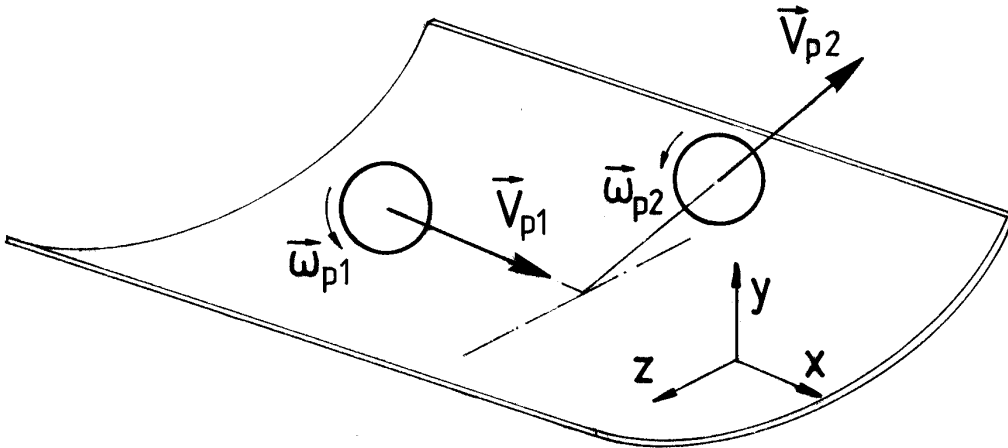


Figure 3.12: Configuration of a particle-wall collision

## 4.2 Wall Roughness Effects

Several experimental studies have shown that wall roughness has a considerable impact on the particle wall collision process (Sommerfeld 1992, Huber & Sommerfeld 1998, Sommerfeld &

Huber 1999). In industrial processes, as for example pneumatic conveying, steel pipes are used, which have a mean roughness height between about 20 and 50  $\mu\text{m}$ . Experimental studies of Sommerfeld & Huber (1999) revealed, that the roughness angle distribution may be represented by a normal distribution function. The standard deviation of this distribution is influenced by the roughness structure and the particle size. The dimensions of the roughness structure suggest, that the wall collision process of small particles (i.e.  $< 100 \mu\text{m}$ ) should be strongly affected, since they will be able to experience the details of the roughness structure (Figure 3.13a)). However, after rebound they will quickly adjust to the flow, so that the influence of the wall roughness effect is limited to the near wall region and will not strongly affect the particle behaviour in the bulk of the flow. On the other hand large particles may cover several roughness structures during wall impact (Figure 3.13b)). This implies that they „feel“ less wall roughness. However, due to their high inertia, they will need more time to adjust to the flow after rebound. This eventually causes the wall roughness to be more important for the bulk behaviour of larger particles in a given flow (Sommerfeld 1992 and 1996).

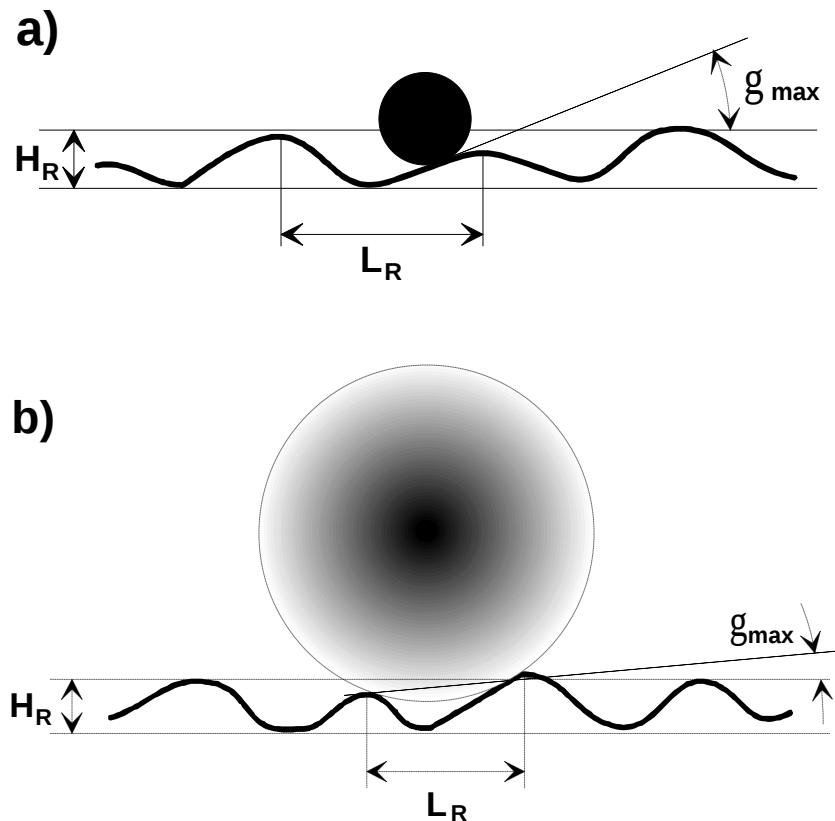


Figure 3.13: Illustration of wall roughness effect for small and large particles

In addition, the so-called shadow effect for small impact angles results in a shift of the effective roughness angle distribution towards positive values, since the particles are not able to reach the lee-side of the roughness structures (Fig. 3.14). Hence, for small impact angles the effective impact angle is increased compared to the particle trajectory angle with respect to the plane wall (Sommerfeld & Huber 1999). This implies a transfer of momentum from the wall-parallel component to the normal component, i.e. the normal component of the rebound velocity becomes larger than the impact component (Figure 3.15). In pneumatic conveying this effect causes a re-dispersion of the particles, whereby the influence of gravitational settling is reduced (Huber & Sommerfeld 1998)

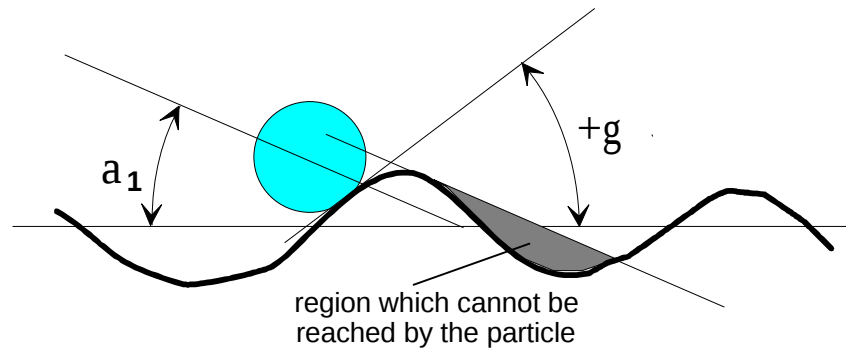


Figure 3.14: Illustration of the shadow-effect for small impact angles

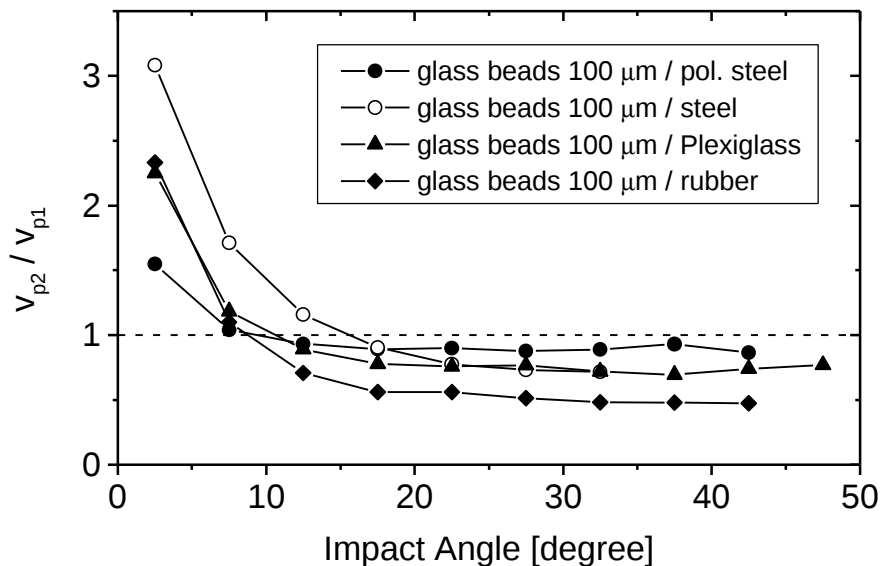


Figure 3.15: Measured dependence of the velocity ratio for the component normal to the wall on the impact angle for different wall material and 100  $\mu\text{m}$  glass beads (Sommerfeld & Huber 1999)

### 4.3 Importance of Wall Collisions in Channel and Pipe Flows

In the following section the effect of wall collisions and wall roughness on the behaviour of particles in a horizontal channel of 35 mm height and a length of 6 m is analysed. The gas flow field (i.e. mean velocity and turbulence) was prescribed according to the measurements of Laufer (1952) for a fully developed channel flow with an average velocity of 18 m/s and two-way coupling was not considered. The gas density was given a value of  $1.18 \text{ kg/m}^3$  and the dynamic viscosity was selected to be  $18.8 \cdot 10^{-6} \text{ N s/m}^2$ . The particle motion was calculated by solving the equation of motion including drag force, gravity, transverse lift forces due to shear and particle rotation, and turbulence effects (Sommerfeld et al. 1993). Wall roughness is modelled as proposed by Sommerfeld & Huber (1999), where the effective impact angle is composed of the particle trajectory angle and a stochastic normal distributed component to account for wall roughness. In Figure 3.16 the behaviour of particles with different diameters ( $\rho_p = 2.5 \text{ g/cm}^3$ ) in a channel without and with wall roughness is illustrated. It is obvious that the wall roughness has a stronger effect on the motion of larger particles (compare Figure 3.16 c) and f)), whereby they bounce from wall to wall. The result of the particle trajectory calculation (Figure 3.16) may be summarised by plotting a wall collision Knudsen number over the particle size (Figure 3.17). The wall collision Knudsen number is defined as the mean free path between subsequent wall collisions to the height of the channel. Considering the case without wall roughness it is obvious that small particles are dispersed by turbulence (see also Figure 3.16 a)), whereby the wall collision frequency is relatively low (i.e. the Knudsen number is large). Increasing particle size results in a decrease of the wall collision mean free path and a minimum is obtained for the present flow condition and channel height for a particle size of about  $100 \text{ }\mu\text{m}$ . For larger particles the mean free path of wall collisions again increases, since these heavier particles are stronger affected by gravity, which implies they have a saltating motion without contacting the upper wall (see also Figure 3.16c)). A further increase in particle size again results in a bouncing from wall to wall due to their high inertia (Figure 3.16 d)) and a reduction of the wall collision mean free path (Figure 3.17).

By considering the lift forces described above (see chapters 3.5 and 3.6), the wall collision mean free path is increased due to the action of the lift forces in the direction of the centre of the channel. The consideration of wall roughness changes the picture completely. Small particles are considerably better suspended in the flow (Figure 3.16 e)), while for large particles the bouncing from wall to wall is enhanced (Figure 3.16 f)). A comparison of Figure 3.16 c) and f) suggests that wall roughness causes a re-dispersion of the particles due to the shadow effect as discussed above. This is illustrated in Figure 3.18 where the particle mass

flux profiles for calculations with and without wall roughness are shown. Without wall roughness gravitational settling is observed, while with wall roughness, the particle mass flux is almost constant over the channel height. The particle bouncing from wall to wall results in an almost constant wall collision frequency for particles above about  $200\ \mu\text{m}$ . The Knudsen number of about 10 implies for the considered channel height a wall collision mean free path of 35 cm. For  $30\ \mu\text{m}$  particles the consideration of wall roughness results in a mean free path of about 2 m.

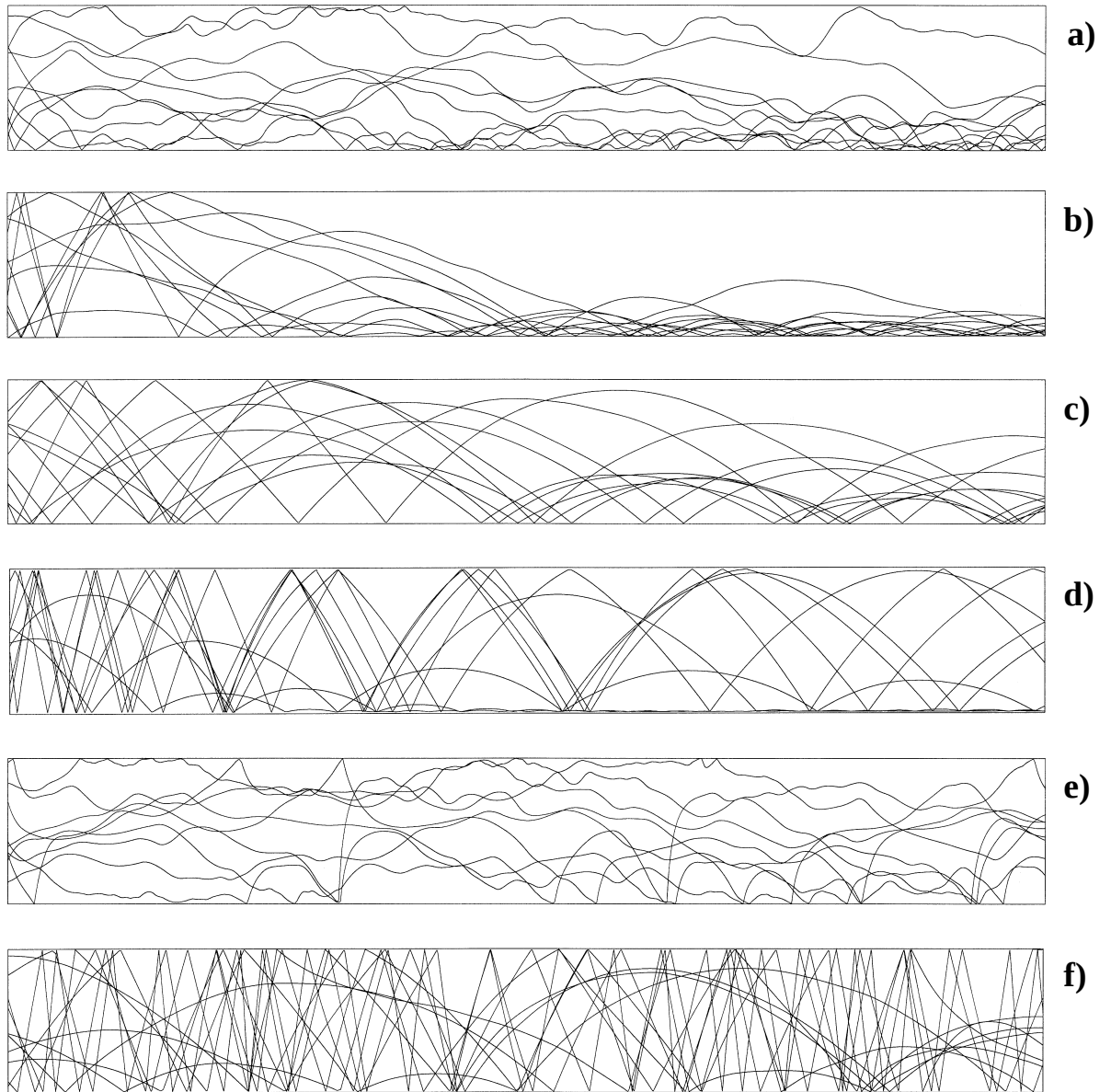


Figure 3.16: Calculated particle trajectories in a horizontal channel flow (channel height 35 mm and length 6 m), without wall roughness: a)  $30\ \mu\text{m}$ , b)  $110\ \mu\text{m}$ , c)  $250\ \mu\text{m}$ , d)  $700\ \mu\text{m}$ , with wall roughness: e)  $30\ \mu\text{m}$ , f)  $250\ \mu\text{m}$  ( $U_{av} = 18\ \text{m/s}$ )

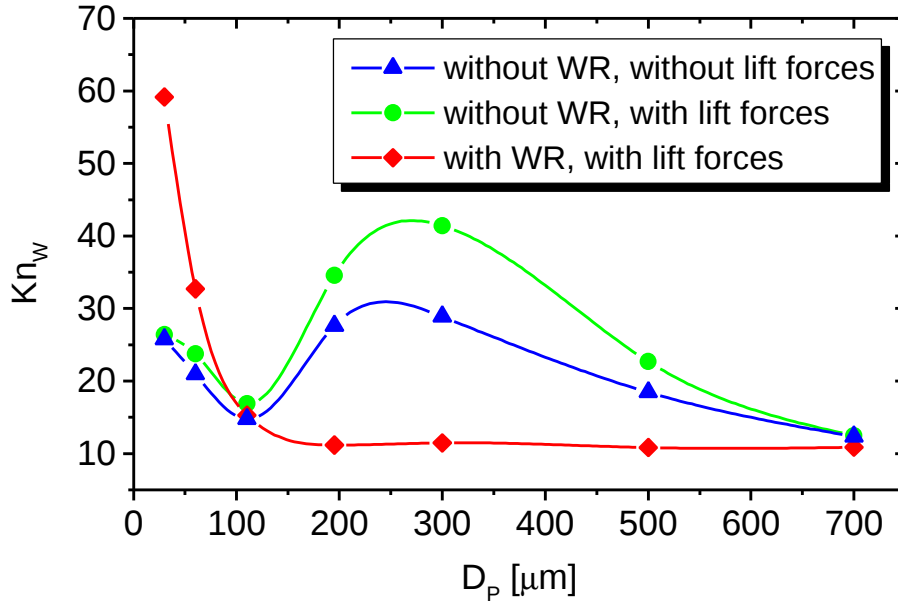


Figure 3.17: Calculated wall collision Knudsen number as a function of particle diameter for a horizontal channel of 35 mm height ( $U_{av} = 18$  m/s)

A detailed analysis of the wall collision process is an interesting topic of research, but eventually the process design is dominated by the operational cost, which means in case of pneumatic conveying the pressure loss is of great importance. The overall pressure loss in pneumatic conveying is dominated by the particle phase and especially wall collisions and roughness may cause a considerable momentum loss for larger particles. This is demonstrated in Figure 3.19, where the calculated streamwise velocity profiles of the particle phase in the horizontal channel are shown. The particle mean velocity is considerably reduced for the case with wall roughness compared to that without and the difference between these two cases is increasing with particle size. Actually, the mean velocity for the 60  $\mu\text{m}$  particles increases in the near wall region, especially near the bottom wall. This is the result of the overall reduction of the wall collision frequency due to wall roughness (see Figure 3.17). The reduction of the particle velocity in the core region of the channel is associated with a stronger dispersion of the particles due to wall roughness (compare Figure 3.16 a) and e)). This discussion suggests, that a considerable increase of pressure loss should result from wall roughness. A numerical computation for a horizontal pipe using the Euler/Lagrange approach (Huber & Sommerfeld 1998) clearly demonstrates this effect. The computed pressure loss along a horizontal pipe increases considerably with wall roughness. In this case a particle size of 40  $\mu\text{m}$  and a

conveying velocity of 18 m/s is considered. The predictions agree reasonably well with the experiments. Further more detailed pressure measurements will be done in the future in order to confirm the results for a smooth wall.

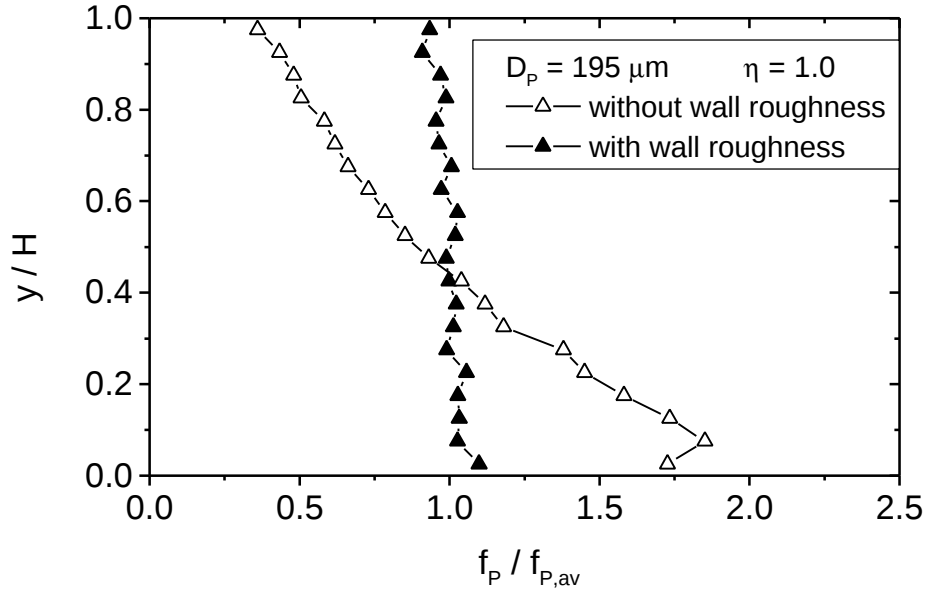


Figure 3.18: Calculated profiles of the particle mass flux in a horizontal channel, influence of wall roughness ( $U_{av} = 18$  m/s,  $195 \mu m$  particle)

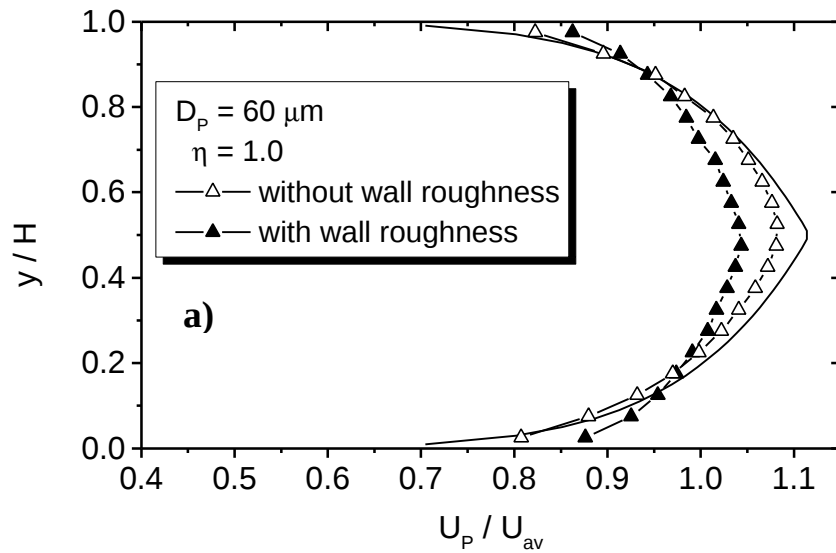


Figure 3.19: (Continued next page)

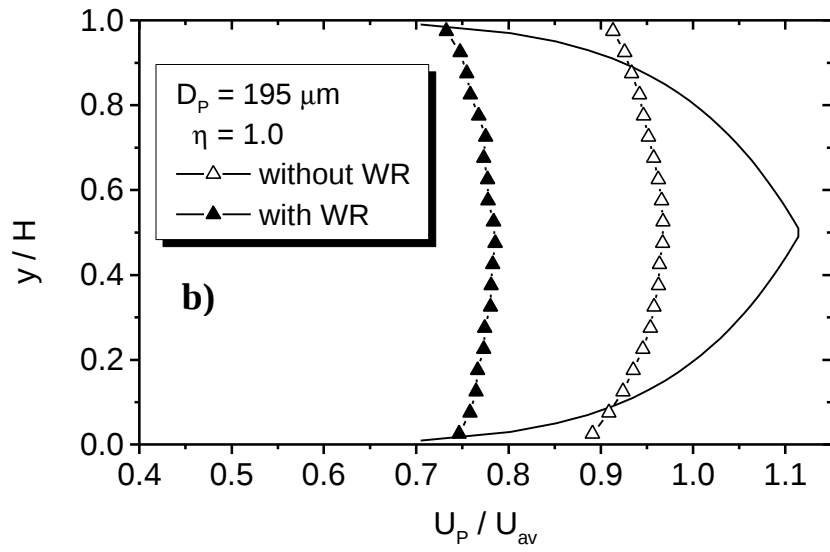


Figure 3.19: Calculated profiles of the particle velocity in a horizontal channel, influence of wall roughness ( $U_{av} = 18$  m/s, closed line represents presumed gas velocity profile), a)  $60 \mu\text{m}$  particle, b)  $195 \mu\text{m}$  particle

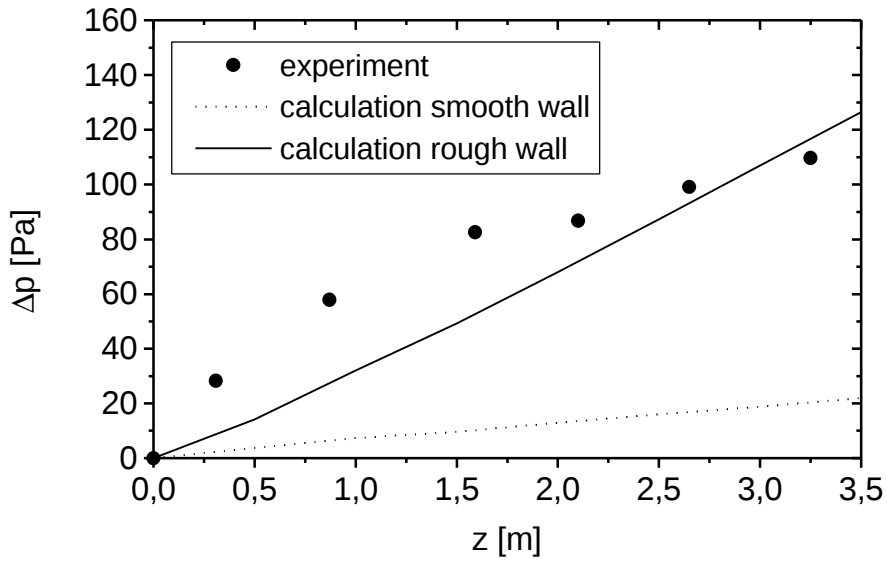


Figure 3.20: Pressure drop along a horizontal pipe with a diameter of 80 mm, calculation using the Euler/Lagrange approach (spherical glass beads with a number mean diameter of  $40 \mu\text{m}$ , conveying velocity 24 m/s, mass loading 0.3 kg particles /kg air), (Huber & Sommerfeld 1998)

## 5. Inter-Particle Collisions

Inter-particle collisions may have several consequences in particle-laden flows, such as heat and momentum transfer between particles, dispersion of regions with locally high particle concentration, and eventually also agglomeration of particles. Essential for inter-particle collisions to occur is a relative motion between the particles. Such a relative motion may be caused by several mechanisms:

- Brownian or thermal motion of particles
- laminar or turbulent fluid shear
- particle inertia in turbulent flow
- mean drift between particles of different size (so-called differential sedimentation)

The collision rate (i.e. collisions per unit volume and time) between two particle fractions  $i$  and  $j$  for **Brownian motion** was given by Smoluchowski (1916) as:

$$N_{ij} = \frac{2 k T}{3 \mu_F} \frac{(D_i + D_j)^2}{D_i D_j} \quad (5.1)$$

where  $D_i$  and  $D_j$  are the diameters of the considered particle classes,  $k$  is the Boltzmann,  $T$  is the absolute temperature constant, and  $\mu_F$  is the dynamic viscosity of the fluid.

For particles which are small compared with the smallest scales of turbulence, i.e. the Kolmogorov length scale, and completely follow the turbulence Saffman and Turner (1956) have provided an expression for the **collision rate due to turbulent shear**:

$$N_{ij} = \left( \frac{8 \pi}{15} \right)^{1/2} n_i n_j (R_i + R_j)^3 \left( \frac{\varepsilon}{\nu_F} \right)^{1/2} \quad (5.2)$$

where  $\varepsilon$  is the dissipation rate of turbulent energy and  $\nu$  is the kinematic viscosity of the fluid.

The **collision rate due to particle inertia in turbulent flow** was also derived by Saffman and Turner (1956) and is given by:

$$N_{ij} = \left( \frac{\pi}{2} \right)^{1/2} \frac{1.3}{18 \mu_F} (D_i + D_j)^2 n_i n_j (\rho_p - \rho_F) (D_i^2 - D_j^2)^2 \left( \frac{\varepsilon^3}{\nu_F} \right)^{1/2} \quad (5.3)$$

The **collision rate due to a mean drift between particles** of different size may be derived from kinetic theory of gases given as:

$$N_{ij} = \frac{\pi}{4} (D_i + D_j)^2 n_i n_j |u_i - u_j| \quad (5.4)$$

Introducing the terminal velocity of the particles, which is for the Stokes-regime obtained from:

$$u_t = \frac{(\rho_p - \rho_F) D_p^2}{18 \mu_F} g \quad (5.5)$$

yields the collision rate for differential sedimentation:

$$N_{ij} = \frac{\pi g}{72 \mu_F} (D_i + D_j)^2 n_i n_j (\rho_p - \rho_F) \cdot |D_i^2 - D_j^2| \quad (5.6)$$

In turbulent flows the particle response and the importance of inter-particle collisions may be characterised by a turbulent Stokes number, which is the ratio of the particle response time  $\tau_p$  to the relevant time scale of turbulence  $T_t$ :

$$St = \frac{\tau_p}{T_t} \quad (5.7)$$

Based on the Stokes number the limiting cases for the collision rate due to turbulence may be identified. For very small particles which completely follow turbulence (i.e.  $St_t \rightarrow 0$ ) the expression of Saffman and Turner (1956), i.e. Eq. 5.2, holds. The other limiting case is the kinetic theory for  $St \rightarrow \infty$ , where the particle motion is completely de-correlated with the fluid and hence the velocity of colliding particles is also de-correlated (i.e. granular medium). This case was analysed by Abrahamson (1975) for heavy particles in high intensity turbulence neglecting external forces, which implies that there is no mean relative velocity between the particles. The resulting collision rate between two particle fractions is given by:

$$N_{ij} = 2^{\frac{3}{2}} \pi^{\frac{1}{2}} n_i n_j (R_i + R_j)^2 \sqrt{\sigma_i^2 + \sigma_j^2} \quad (5.8)$$

where  $\sigma_p$  is the fluctuating velocity of the particles assuming that all components are identical (i.e. isotropic fluctuating motion  $\sigma_p^2 = \overline{u_p'^2} = \overline{v_p'^2} = \overline{w_p'^2}$ ). The expression derived by Abrahamson (1975) is strictly only valid for particles of identical size, since it does not account for a mean drift between the particles. For such a case the collision rate was provided by Gourdel et al. (1999):

$$N_{ij} = \frac{\pi}{4} (D_i + D_j)^2 n_i n_j |U_i - U_j| G(z) \quad (5.9)$$

with:

$$G(z) = \frac{1}{\sqrt{\pi z}} \exp(-z) + \left(1 + \frac{1}{2z}\right) \operatorname{erf} \sqrt{z} \quad (5.10)$$

and:

$$z = \frac{3}{4} \frac{(U_i - U_j)^2}{k_i + k_j} \quad (5.11)$$

In this equation  $U_i$  and  $U_j$  are the mean velocities of the particle fraction  $i$  and  $j$ , and  $k_i$  and  $k_j$  are the energies of their fluctuating velocities namely:

$$k_p = \frac{1}{2} \left( \overline{u_p'^2} + \overline{v_p'^2} + \overline{w_p'^2} \right) \quad (5.12)$$

In practical two-phase flows the two limits (i.e particles completely following turbulence ( $St_t \rightarrow 0$ ) and heavy particles ( $St_t \rightarrow \infty$ )) are rarely met, rather the particles may partially respond to turbulence. Hence, the velocities of colliding particles will be correlated to a certain degree, since they are transported in the same turbulent eddy upon collision. The degree of correlation depends on the turbulent Stokes number defined above (Eq. 5.7). An analysis of this effect was performed by Williams and Crane (1983) and an analytic expression for the collision rate of particles in turbulent flows covering the entire range of particle Stokes numbers and accounting for a possible correlation of the velocities of colliding particles was suggested. The expression for the collision rate is given in terms of particle concentration, particle relaxation times (i.e. Stokes numbers), turbulence intensities, and turbulent scales:

$$N_{ij} = (162 \pi)^{\frac{1}{2}} n_i n_j v_F L_t \frac{\rho_F}{\rho_p} \frac{\overline{u_{ij}}}{\sigma_F} (St_i^{0.5} + St_j^{0.5})^2 \cdot \frac{2}{\pi} \tan^{-1} \left\{ \frac{1}{3} \frac{\rho_p}{\rho_F} \frac{\sigma_F L_t}{v_F} \left( \frac{\overline{u_{ij}}}{\sigma_F} \right)^2 \frac{St_i St_j}{(St_i^{0.5} + St_j^{0.5})^2} \right\} \quad (5.13)$$

Here  $L_t$  is the integral length scale of turbulence,  $\overline{u_{ij}}$  the mean relative velocities between colliding particles, and  $\sigma_F$  the fluctuating velocity of the fluid assuming isotropic turbulence. The Stokes numbers of the two particle classes  $St_i$  and  $St_j$  are defined in terms of the integral time scale of turbulence.

### 5.1 Importance of Inter-Particle Collisions

In the following the importance of inter-particle collisions in turbulent fluid-particle flows is discussed. The inter-particle collision probability depends mainly on the particle concentration, the particle size, and the fluctuating motion of the particles. A classification of particle-laden flows in terms of the importance of inter-particle collisions and the boundary between dilute and dense systems may be based on the ratio of particle response time  $\tau_p$  to the averaged time between collisions  $\tau_c$  (Crowe 1981). In dilute two-phase flows the particle motion will be mainly governed by fluid dynamic transport effects, i.e. drag force, lift forces, and turbulence. On the other hand dense flows are characterised by high collision frequencies between particles and hence their motion is dominantly influenced by inter-particle collisions.

Fluid dynamic transport effects are of minor importance. Therefore, the two regimes are characterised by the following time scale ratios:

- **dilute two-phase flow:**

$$\frac{\tau_p}{\tau_c} < 1 \quad (5.14)$$

- **dense two-phase flow:**

$$\frac{\tau_p}{\tau_c} > 1 \quad (5.15)$$

This implies, that in dense two-phase flows the time between particle-particle collisions is smaller than the particle response time, whereby the particles are not able to completely respond to the fluid flow between successive collisions. This regime may occur when either very large particles at a low number density are present in the flow or in the case of small particles when the number density is large. In dilute two-phase flows collisions between particles may also occur and influence the flow development to a certain degree, but the time between successive inter-particle collisions is larger than the particle response time, whereby the fluid dynamic transport of the particles is the dominant transport effect.

In the following section an estimate of the boundary between the two regimes will be given for turbulent particle-laden flows. The average time between successive inter-particle collisions results from the average collision frequency:

$$\tau_c = \frac{1}{f_c} \quad (5.16)$$

The collision frequency of one particle (i.e.  $n_i = 1$ ) with diameter  $D_i$  and velocity  $\bar{u}_i$  with all other particle classes (i.e.  $N_{\text{class}}$ ) with diameter  $D_j$  and velocity  $\bar{u}_j$  can be calculated according to kinetic theory of gases from:

$$f_c = \frac{N_{ij}}{n_i} = \sum_{j=1}^{N_{\text{class}}} \frac{\pi}{4} (D_i + D_j)^2 |\bar{u}_i - \bar{u}_j| n_j \quad (5.17)$$

The main assumptions associated with the use of Eq. 5.17 are the following:

- The particle number concentration is small enough that the occurrence of binary collisions prevails.
- On the other hand the particle number concentration must be large enough to allow a statistical treatment.
- The velocities of the colliding particles are not correlated.

An analytic solution of Eq. 5.17 is only possible for relatively simple cases. For the estimation of the collision frequency, the derivation of Abrahamson (1975) is followed, yielding a collision rate solely determined by turbulence (Eq. 5.8). Furthermore, a mono-disperse particle phase is considered, whereby the mean fluctuating velocity is a constant. Hence the collision frequency is obtained as a function of the particle diameter  $D_p$ , the total particle number concentration  $n_p$  and the mean fluctuating velocity of the particles  $\sigma_p$ :

$$f_c = 4 \pi^{1/2} n_p D_p^2 \sigma_p \quad (5.18)$$

Introducing the volume fraction of the particles  $\alpha_p = (\pi/6) D_p^3 n_p$ , one obtains after some rearrangements:

$$f_c = \frac{24}{\pi^{1/2}} \frac{\alpha_p \sigma_p}{D_p} \quad (5.19)$$

or similarly the collision frequency can be expressed as a function of the mass loading  $\eta = \dot{m}_p / \dot{m}_F$ , which is often used to characterise gas-solid flows:

$$f_c = \frac{24}{\pi^{1/2}} \frac{\rho_F}{\rho_p} \frac{\eta \sigma_p}{D_p} \quad (5.20)$$

By introducing now the collision frequency and the Stokesian particle response time into Eqs. 5.19 and 5.20, the limiting particle diameter for a dilute two-phase flow can be determined as a function of volume fraction or mass loading, respectively:

$$\begin{aligned} D_p &< \frac{3}{4} \pi^{1/2} \frac{\mu_F}{\alpha_p \rho_p \sigma_p} \\ D_p &< \frac{3}{4} \pi^{1/2} \frac{\mu_F}{\eta \rho_F \sigma_p} \end{aligned} \quad (5.21)$$

Considering a gas-solid flow with the properties ( $\rho_F = 1.15 \text{ kg/m}^3$ ,  $\rho_p = 2500 \text{ kg/m}^3$ ,  $\mu_F = 18.4 \cdot 10^{-6} \text{ kg/(m s)}$ ) the limiting particle diameter which separates dilute and dense two-phase flow is calculated as a function of volume fraction and mass loading with the particle velocity fluctuation as a parameter. The result is given in Figure 5.1 where the dilute two phase flow is the domain left of the individual lines and the dense flow regime is on the right hand side. With increasing particle diameter associated with higher particle inertia, the range of dilute flow is shifted towards lower volume fractions and mass loading. With increasing velocity fluctuation of the particles the boundary line between dilute and dense two-phase flow is shifted to the left, i.e. to smaller mass loading of the dispersed phase. From Eq. 5.18 it is obvious that the collision frequency increases with the velocity fluctuation of the particles.

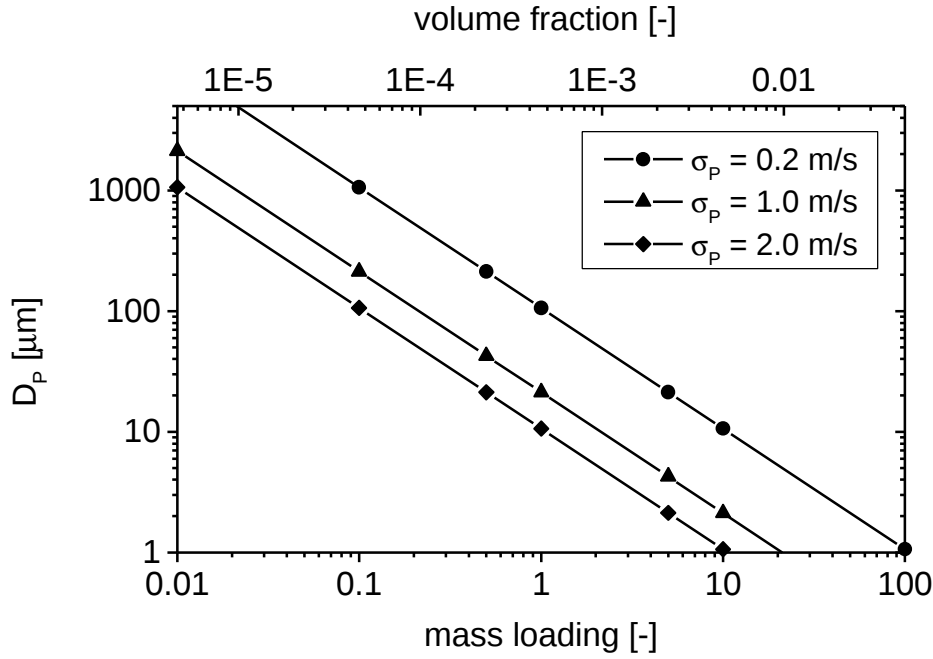


Figure 5.1: Regimes of dilute and dense gas-solid flows in dependence of mass loading (and volume fraction) and particle diameter for different particle velocity fluctuations ( $\rho_F = 1.15 \text{ kg/m}^3$ ,  $\rho_p = 2500 \text{ kg/m}^3$ ,  $\mu_F = 18.4 \cdot 10^{-6} \text{ kg/(m s)}$ )

## 5.2 Particle Velocity Change due to Inter-Particle Collisions

The calculation of the particle velocity change by an inter-particle collision relies generally on the following assumptions:

- Only binary collisions are considered, which is valid for most dispersed two-phase systems.
- The deformation of the particles during the collision process is neglected, which results in the so-called hard sphere model.
- The friction in the case of a sliding collision is described by Coulomb's law of friction.

Hence, the change of linear and angular velocity components can be calculated by solving the momentum equations in connection with Coulomb's law of friction, so that again relations are obtained for a sliding and non-sliding collision. The problem may be further simplified by transforming the particle velocities into a coordinate system where one of the particles is stationary, for example particle ② in Figure 5.2. For such a collision geometry, where the relative velocity vector coincides with the axis of the collision cylinder, the relations for the calculation of the post-collision properties of both particles reduce to that for an oblique central collision (Oesterle & Petitjean 1994, Sommerfeld 1995). Hence, one obtain the

following set of equations to calculate the new linear and angular velocity components of both particles in terms of the relative velocity components before collision:

$$\begin{aligned} u_{p1}^* &= u_{p1} + \frac{J_x}{m_{p1}} & u_{p2}^* &= -\frac{J_x}{m_{p2}} \\ v_{p1}^* &= v_{p1} + \frac{J_y}{m_{p1}} & v_{p2}^* &= -\frac{J_y}{m_{p2}} \\ w_{p1}^* &= +\frac{J_z}{m_{p1}} & w_{p2}^* &= -\frac{J_z}{m_{p2}} \end{aligned} \quad (5.22)$$

$$\begin{aligned} \omega_{p1}^{*x} &= \omega_{p1}^x & \omega_{p2}^{*x} &= \omega_{p2}^x \\ \omega_{p1}^{*y} &= \omega_{p1}^y - \frac{5 J_z}{m_{p1} D_{p1}} & \omega_{p2}^{*y} &= \omega_{p2}^y + \frac{5 J_z}{m_{p2} D_{p2}} \\ \omega_{p1}^{*z} &= \omega_{p1}^z + \frac{5 J_y}{m_{p1} D_{p1}} & \omega_{p2}^{*z} &= \omega_{p2}^z - \frac{5 J_y}{m_{p2} D_{p2}} \end{aligned} \quad (5.23)$$

Here,  $m_{p1}$  and  $m_{p2}$  are the masses of both particles and  $J_x$ ,  $J_y$  and  $J_z$  are the components of the impulsive force. With the definition of the normal restitution ratio:

$$e = -\frac{u_{p1}^* - u_{p2}^*}{u_{p1}} \quad (5.24)$$

and the conservation of the x-component of the momentum for particle 2:

$$J_x = -m_{p2} u_{p2}^* \quad (5.25)$$

one finally obtains the following expression for  $J_x$ :

$$J_x = -(1+e) u_{p1} \frac{m_{p1} m_{p2}}{m_{p1} + m_{p2}} \quad (5.26)$$

By applying Coulomb's law of friction one obtains furthermore the condition for a non-sliding collision as a function of the static coefficient of friction  $\mu_0$ :

$$\sqrt{J_y^2 + J_z^2} < \mu_0 |J_x| \quad (5.27)$$

Now the components of the impulse force are introduced into Eq. 5.27 and the condition for a non-sliding collision is obtained in dependence on the velocities of both particles before collision.

$$|u_R| < \frac{7}{2} \mu_0 (1+e) |u_{p1}| \quad (5.28)$$

The relative velocity at the point of contact is determined with the linear and angular velocity components of both particles.

$$\begin{aligned}
u_R &= \sqrt{u_{Ry}^2 + u_{Rz}^2} \\
u_{Ry} &= v_{p1} + \frac{D_{p1}}{2} \omega_{p1}^z + \frac{D_{p2}}{2} \omega_{p2}^z \\
u_{Rz} &= -\frac{D_{p1}}{2} \omega_{p1}^y - \frac{D_{p2}}{2} \omega_{p2}^y
\end{aligned} \tag{5.29}$$

The components of the impulsive force  $J_y$  and  $J_z$  are dependent on the type of collision. For a **non-sliding collision** one obtains:

$$\begin{aligned}
J_y &= -\frac{2}{7} u_{Ry} \frac{m_{p1} m_{p2}}{m_{p1} + m_{p2}} \\
J_z &= -\frac{2}{7} u_{Rz} \frac{m_{p1} m_{p2}}{m_{p1} + m_{p2}}
\end{aligned} \tag{5.30}$$

and for a **sliding collision** the components of the impulsive force are dependent on the dynamic coefficient of friction  $\mu_d$ :

$$\begin{aligned}
J_y &= -\mu_d \frac{u_{Ry}}{u_R} |J_x| \\
J_z &= -\mu_d \frac{u_{Rz}}{u_R} |J_x|
\end{aligned} \tag{5.31}$$

Once the new velocities are obtained they are re-transformed into the original co-ordinate system. The above equations show that the parameters involved in the collision model are the restitution coefficient  $e$ , and the static and dynamic coefficient of friction.

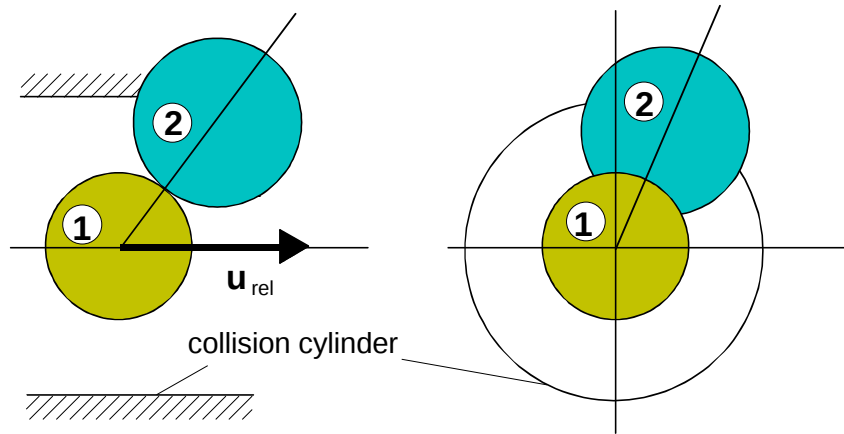


Figure 5.2: Particle-particle collision configuration and co-ordinate system

Modelling of inter-particle collisions in the frame of the Euler/Lagrange method for the numerical calculation of two-phase flows has been based mainly on two approaches, a direct simulation and a model based on concepts of the kinetic theory of gases. The most straight forward approach to account for inter-particle collisions is the direct simulation approach.

This requires that all the particles are tracked simultaneously through the flow field. Thereby, the occurrence of collisions between any pair of particles can be judged based on their positions and relative motion during one time step. Once a collision occurs the change in translational and angular particle velocities can be determined by solving the above equations. This approach is however very time consuming, since at each time step it requires to search for a possible collision partner among the surrounding particles (see for example Tanaka and Tsuji 1991).

Therefore, based on previous work by Oesterle & Petitjean (1993) and Sommerfeld & Zivkovic (1992) a stochastic inter-particle collision model was developed, which also accounts for the velocity correlation of colliding particles and is much more efficient (Sommerfeld 2000). This stochastic inter-particle collision model is briefly described in the following:

- At each time step of the trajectory calculation a fictitious collision partner is generated with size and velocity sampled from local distribution functions.
- In sampling the fictitious particle velocity a possible correlation due to turbulence is respected.
- The collision probability (i.e. the product of collision frequency and time step) is calculated on the basis of Eq. 5.4. If a uniform random number in the interval between zero and one becomes smaller than this probability a collision is calculated.
- By transforming the particle velocities into a coordinate system where the fictitious particle is stationary, it is possible to sample the point of contact which only can be located on the hemisphere facing the fictitious particle.
- The new velocities of the considered particle are calculated based on the equations introduced above (i.e. Eqs. 5.22 to 5.31).
- Finally, the particle velocities are re-transformed into the original coordinate system. The fictitious particle is not of further interest.

Fluid dynamic effects during the collision process may be neglected if the duration of the collision process is negligibly small compared to the time of collisionless motion, the size of the colliding particles is not too different, and the ratio of solid particle density to the fluid density is much larger than unity. Hence under such conditions the collision efficiency may be assumed to be 100 %.

### 5.3. Inter-particle Collision Effects in Turbulent Flows

The following results were obtained with the stochastic inter-particle collision model briefly described above. The first case considered is a cube with homogeneous isotropic turbulence without gravity, which was analysed in detail by Lavieville et al. (1995) using large eddy simulations (LES). These results served a test case for validating the stochastic collision model. Figure 5.3 shows the collision frequency obtained from the model calculations for a wider range of Stokes numbers. As expected, the uncorrelated model predicts a continuous increase of the collision frequency with decreasing Stokes number. The correlated model however, predicts a maximum in the collision frequency for a Stokes number of about 0.4. For smaller Stokes numbers a decrease of the collision frequency is found and in the limit of particles completely following the turbulent fluctuations (i.e.  $St_t \rightarrow 0$ ) the value predicted by Saffman and Turner (1961) is approached. Hence, the developed correlated collision model correctly predicts the collision rates in isotropic homogeneous turbulence (see Sommerfeld 1999 and 2000 for further results).

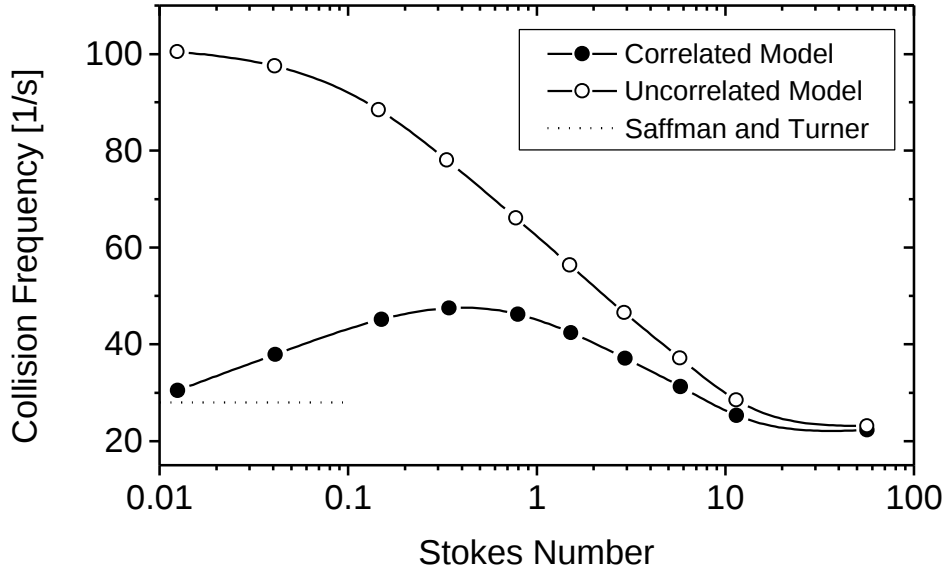


Figure 5.3: Dependence of the ratio of simulated collision frequency to the collision frequency obtained from the kinetic theory limit on the particle Stokes number ( $\alpha_p = 0.0176$ )

In the second case a binary mixture of particles with the same size, but different density, settling under the action of gravity (i. e.  $g_x = 49.05 \text{ m/s}^2$ ) in a cube with homogeneous isotropic turbulence is considered (Sommerfeld 2000). In this test case the collisions between particles are mainly determined by the mean drift between the two particle fractions as a result

of their different terminal velocity. In addition collisions are induced by the fluctuating motion of the particles, which is partly caused by turbulence. Hence, the collisions between the two particle fractions causes a momentum transfer between the two fractions, whereby the mean velocity (i.e. in the direction of gravity) of the light particles (fraction A) becomes larger than their terminal velocity and for the heavy particles the settling is hindered by collisions with the light particles (Figure 5.4). At low volume fractions of class B particles, the heavy particles are strongly hindered by the light ones and hence the heavy particle mean velocity is about 19 % smaller than their terminal velocity. With increasing volume fraction of class B, the heavy particles drag the light particles and their mean velocity increases, while the mean velocity of the heavy particles also increases and approaches the expected terminal velocity. These effects are well reproduced in the model calculations and the agreement with the LES results is reasonably good.

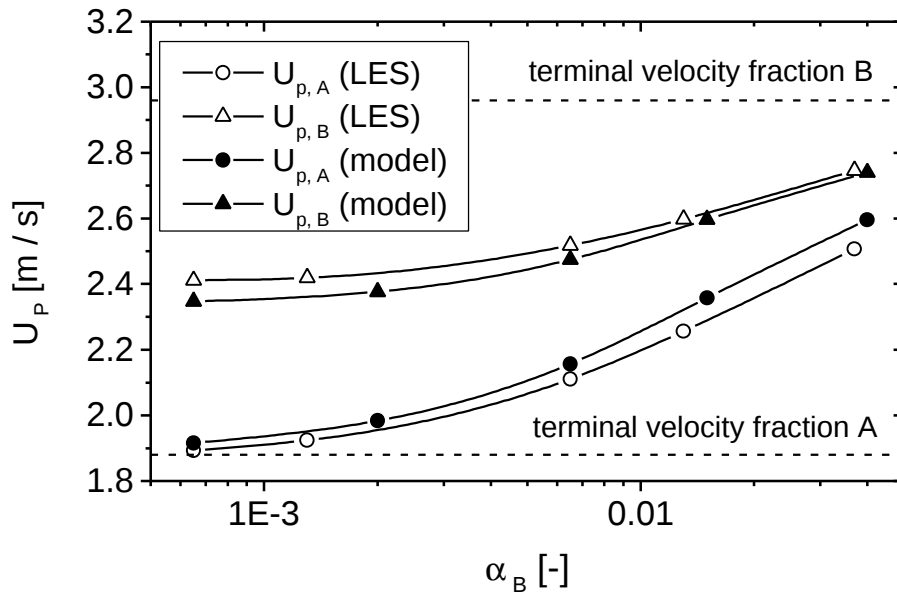


Figure 5.4: Mean particle velocities for fraction A and B, comparison of model calculations with LES data for a binary mixture settling under gravity ( $\alpha_p = 0.013$ )

Considering the collision frequencies for this case (Figure 5.5), it is found that  $f_{AA}$  increases with the concentration of fraction B due to an increase of the fluctuating motion of fraction A. The collision frequencies between fraction A and B (i.e.  $f_{AB}$ ) increases mainly due to the increase of the volume fraction of the particles fraction B. The agreement of the model calculations with the LES-results is reasonably good for this case.

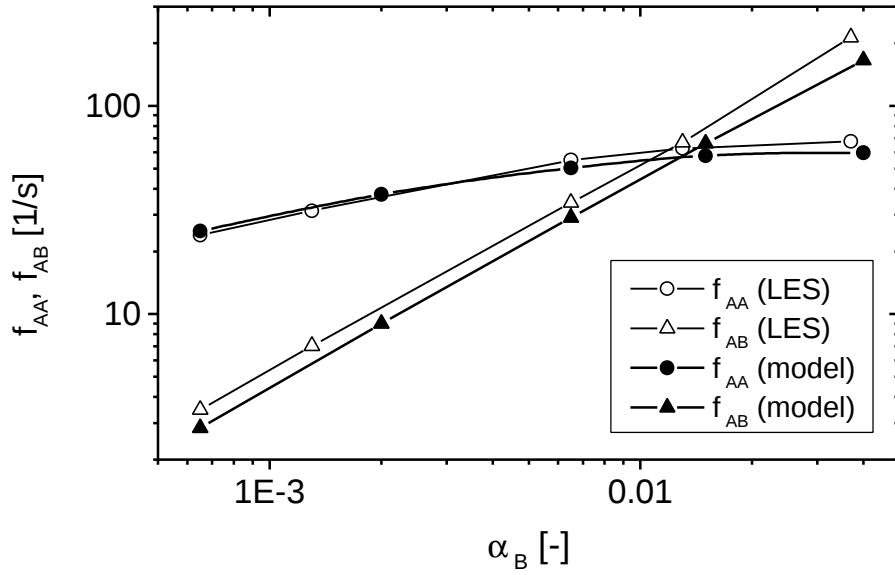


Figure 5.5: Collision frequencies for fraction A and between fraction A and B, comparison of model calculations with LES data for a binary mixture settling under gravity

For analysing the influence of inter-particle collisions on the development of a particle-laden gas flows, a horizontal channel with a height of  $H = 50$  mm and a length of  $L = 5$  m was considered. In order to isolate the effects of particle collisions from other effects, two-way coupling was not considered and the gas flow field was prescribed according to the measurements of Laufer (1952) for a fully developed channel flow with a centre line velocity of 20 m/s. The gas density was given a value of  $1.18 \text{ kg/m}^3$  and the dynamic viscosity was selected to be  $18.8 \cdot 10^{-6} \text{ N s/m}^2$ . The particle motion was calculated by solving the equation of motion including drag force, gravity, transverse lift forces due to shear and particle rotation, and turbulence effects (Sommerfeld et al. 1993). The wall collision of particles was treated as inelastic and the change of the velocities was calculated by solving the equations introduced in Chapter 4.1. Wall roughness however, was not considered these calculations. The particles considered in the calculations were glass beads with different diameters in the range between 25 and 500  $\mu\text{m}$  and a solids density of  $\rho_p = 2.5 \text{ g/cm}^3$ .

The effect of inter-particle collisions is analysed by first comparing the particle mass flux distributions for different particle sizes as a function of mass loading. For the smallest particles considered (i.e. 25  $\mu\text{m}$ ) the influence of inter-particle collisions on the mass flux profile is very small and the profiles only slightly change with increasing mass loading (Figure 5.6 a)), indicating that a particle re-distribution is hindered due to the high collision frequencies resulting from the large number concentration of the particles compared to the

larger particles and their fast response to the fluid flow. With increasing particle size, gravitational settling becomes more important and the particle mass flux near the bottom of the channel becomes much larger than for small particles without inter-particle collisions (Figure 5.6 b)). Inter-particle collisions result in a re-dispersion of the particles and with increasing mass loading and hence increasing collision frequency the mass flux near the bottom of the channel is reduced. At the highest mass loading (i.e.  $\eta = 2.0$ ) the particles are almost homogeneously dispersed over the channel and the maximum in the mass flux is found at some distance above the channel bottom.

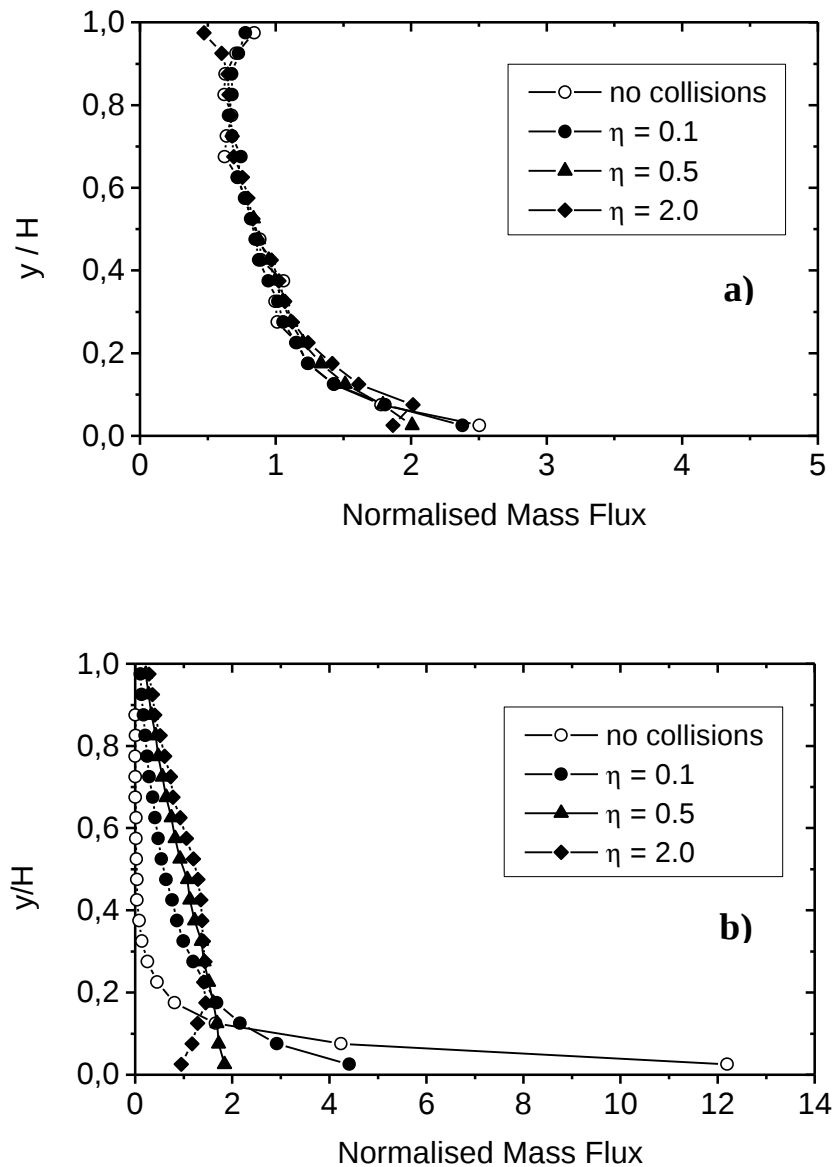


Figure 5.6: Influence of inter-particle collisions on the developed profiles of the normalised particle mass flux for different mass loading and particle sizes, a)  $D_p = 25 \mu m$ , b)  $D_p = 100 \mu m$  ( $U_0 = 20 \text{ m/s}$ )

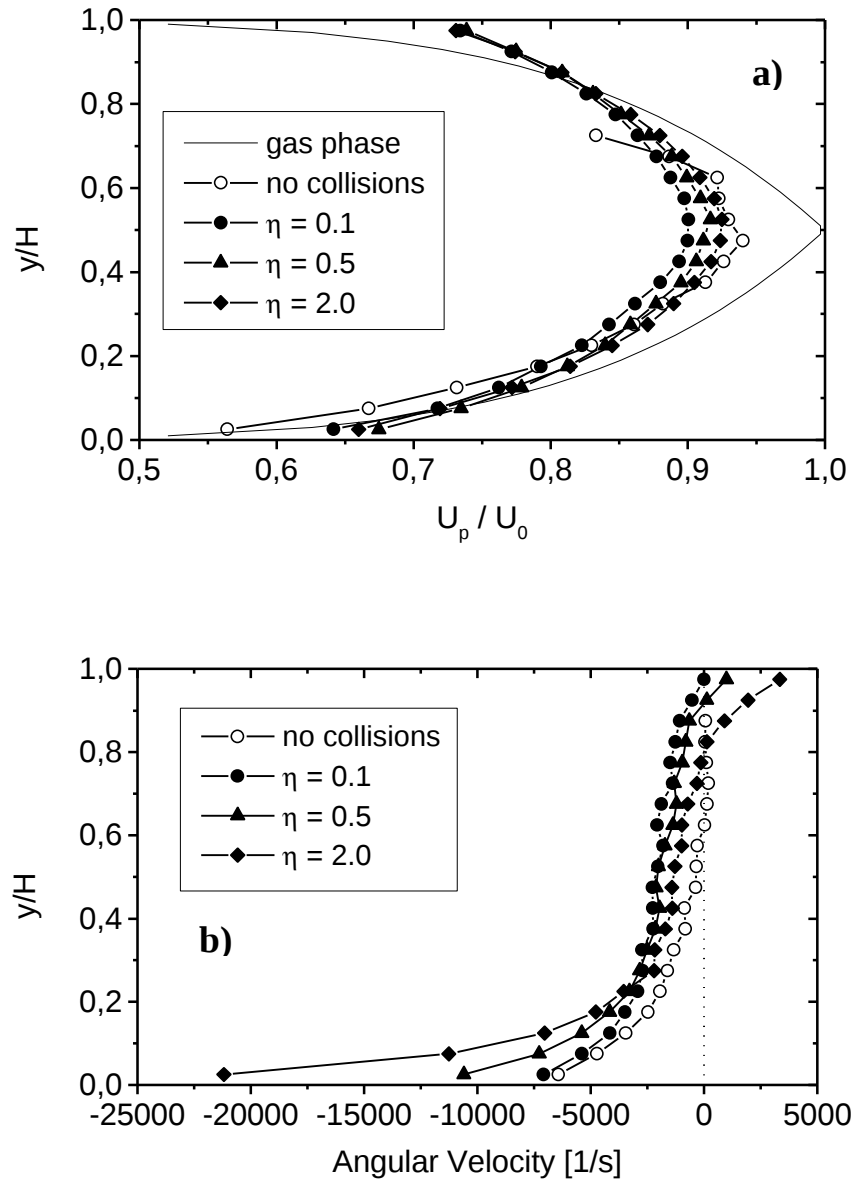


Figure 5.7: Influence of inter-particle collisions on the profiles of the particle velocities, a) stream-wise component of the velocity (the closed line represents the prescribed gas velocity), b) angular velocity, ( $D_p = 100 \mu\text{m}$ ,  $U_0 = 20 \text{ m/s}$ )

The influence of inter-particle collisions on the profiles of the linear and angular velocities is illustrated in Figure 5.7 for the  $100 \mu\text{m}$  particles. Even for a rather low mass loading of  $\eta = 0.1$ , the horizontal component of the particle mean velocity is considerably affected by inter-particle collisions. Near the bottom of the channel, the particle velocity becomes larger and in the core region smaller compared to the result without taking account of collisions. This is caused by an enhancement of transverse particle transport due to inter-particle collisions (see also Figure 5.8 b)) whereby low-speed particles from the region of high concentration near the

bottom of the channel are ejected into the core region and high-speed particles from the core region bounce back towards the wall. With further increase of the particle mass loading, i.e. from  $\eta = 0.1$  to 2.0, the collision frequency between particles is increased and the particle concentration near the bottom of the channel is further reduced. This also is associated with a further reduction of the wall collision frequency. Thereby, the particles lose less momentum and hence the particle mean velocity in the core region of the channel is again slightly increasing with mass loading. The angular velocity of the particles, especially in the vicinity of the channel bottom, is also considerably enhanced by the collisions between the particles (Figure 5.7 b)). For a mass loading of 2.0 the angular velocity reaches values of more than 20,000 1/s near the bottom of the channel.

Considering the fluctuating velocity components, significant effects of inter-particle collisions are obvious from Figure 5.8. The stream-wise component is first of all significantly higher than that of the gas phase and moreover, higher in the lower part of the channel than in the upper part (Figure 5.8 a)). This is again provoked by wall collision effects and the higher wall collision frequency with the lower wall. Obviously, the collisions between particles enhance the stream-wise fluctuation of the particles initially, when comparing the results without and with accounting for collisions at a loading ratio of 0.1. A further increase of the mass loading, being connected with an increasing inter-particle collision frequency, causes a decrease of the particle fluctuating velocity for the horizontal component. On the other hand the transverse component of the particle fluctuating velocity is continuously increasing with higher particle collision frequencies (Figure 5.8 b)). Hence, the particles fluctuating motion becomes more isotropic as a result of inter-particle collisions and the associated momentum transfer. These results reveal that whenever inhomogenities of the particle concentration develop in a flow, inter-particle collisions become of great importance already for quite low overall particle concentration. This effect was recently also observed for a quite important process in chemical engineering, namely the dispersion of particles in a stirred vessel (Decker & Sommerfeld 2000).

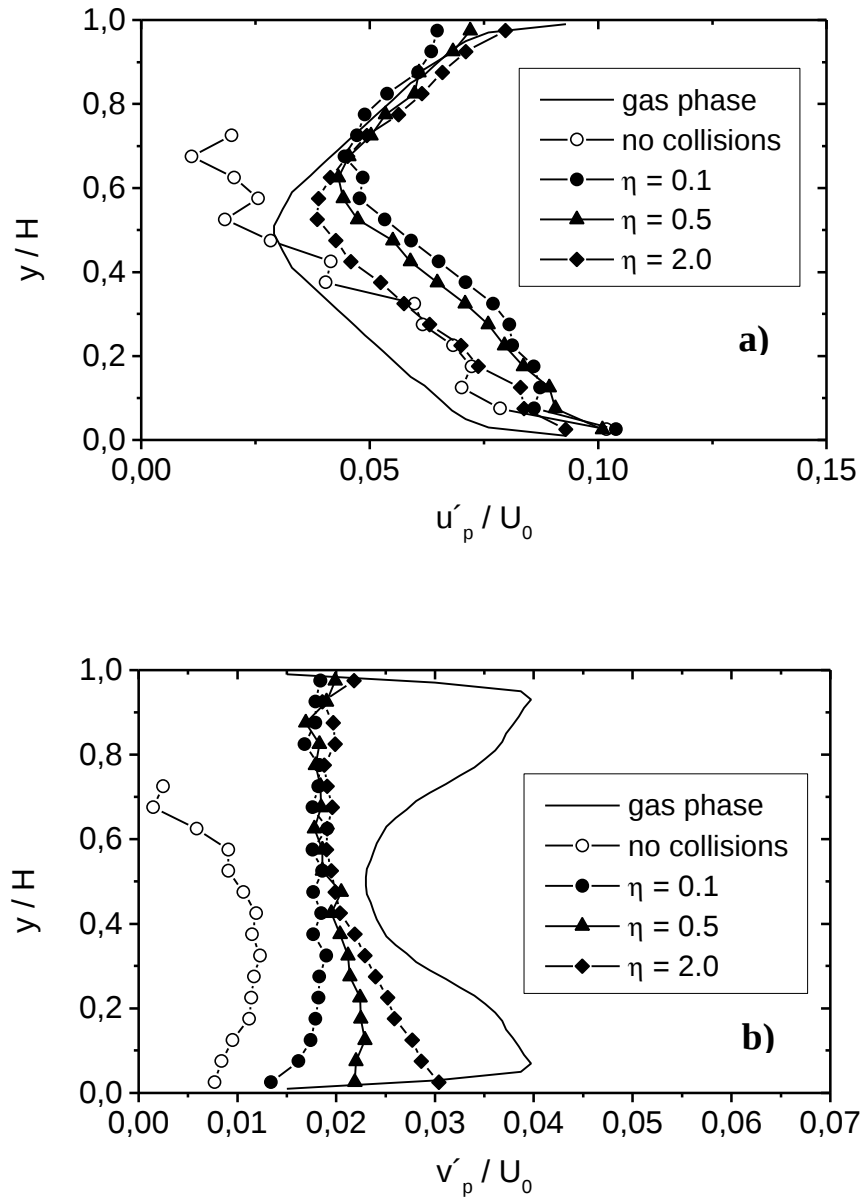


Figure 5.8 Influence of inter-particle collisions on the profiles of the fluctuating velocities of the particles, a) stream-wise component, b) lateral component (particle diameter:  $100 \mu\text{m}$ ,  $U_0 = 20 \text{ m/s}$ , closed lines indicate the prescribed profiles of the gas phase mean fluctuating velocities)

## 6. Methods for the Prediction of Multiphase Flows

Over the past 10 years computational fluid dynamics (CFD) is increasingly used by chemical industry for process analysis and optimisation. Most of these processes involve single- or multi-phase flows in complex geometries which may be also accompanied by heat and mass transfer and chemical reactions. Numerical computations of multiphase flows may be

performed on different levels of complexity related to the resolution of the interface between the phases and the turbulence modelling:

- Direct numerical simulations of particulate flows by accounting for the finite dimensions of the particles and the flow around the particles have become feasible in the past couple of years due to the drastic increase of computational power. Such an approach the time-dependent solution of the three-dimensional Navier-Stokes equations on a grid which resolves the particles. Two approaches are mainly being used to resolve the particle contour and respect the appropriate boundary condition at the surface. An adaptive unstructured grid is used in order to follow the particle motion and resolve the particle shape (see for example Hu (1996)). In the second approach the flow field is calculated on a regular grid and a tracer particle method (i.e. a template) is used to emulate the rigid particle. The coupling with the fluid is achieved through a volume force term in the Navier-Stokes equation See for example Schwarzer et al. 1998, and Glowinski et al. 1999). Mainly these kind of methods have been applied to flows with low particle Reynolds number, e.g. to calculate particle sedimentation processes.
- Direct numerical simulations (DNS), as described above, are also being applied to dispersed turbulent two-phase flows by considering the particles as point-particles and using the Lagrangian approach to simulate the dispersed phase. This implies that a large number of particles are simultaneously tracked through the computed time-dependent flow field by considering the relevant forces. This type of DNS has been applied mainly to basic turbulence research, in order to analyse the particle behaviour in turbulent flows (Squires & Eaton 1990, Elghobashi & Truesdell 1992), and the effect of particle on turbulence (Elghobashi & Truesdell 1993, Squires & Eaton 1994).
- Large eddy simulations combined with the Lagrangian tracking of point-particles have been also applied to the study of basic phenomena in dispersed particulate flows, such as, particle dispersion in turbulent flows, inter-particle collisions (Lavieville et al. 1995) and particle behaviour in channel flows (Wang et al. 1997). The results are mainly used to derive and validate models and also to develop closures for the two-fluid approach.
- For engineering problems two approaches based on the Reynolds-averaged Navier-Stokes equations are commonly applied, namely the two-fluid or Euler/Euler approach and the Euler/Lagrange method. In order to account for the interaction between phases, i.e. momentum exchange and heat and mass transfer, the conservation equations have to be extended by appropriate source/sink terms.

In the two-fluid approach both phases are considered as interacting continua. Hence, properties such as the mass of particles per unit volume are considered as a continuous property and the particle velocity is the averaged velocity over an averaging volume (i.e. computational cell). Also the interfacial transfer of mass, momentum, or energy requires averaging over the computational cells. Especially in turbulent flows the closure of the dispersed phase Reynolds-stresses and the fluid-particle interaction terms are associated with sophisticated modelling approaches (see e.g. Rizk & Elghobashi (1989) and Simonin et al. 1993). The consideration of a particle size distribution requires the solution of a set of basic equations for each size class to be considered. Hence the computational effort increases with the number of size classes. The method is however preferable for discontinuous and dense two-phase flows, as for example found in fluidised beds (Blazer & Simonin 1993). A detailed review on recent developments of Eulerian models for the prediction of fluidisation processes was recently published by Enwald et al. (1996).

The Euler/Lagrange approach is only applicable to dispersed two-phase flows and accounts for the discrete nature of the individual particles. The dispersed phase is modelled by tracking a large number of particles through the flow field in solving the equations of motion taking into account the relevant forces acting on the particle. Generally, the particles are considered as point-particles, i.e. the finite dimension of the particles is not considered and the flow around the individual particles is not resolved. Since the number of real particles in a flow system is usually too large for allowing a tracking of all particles, the trajectories of computational particles (i.e. parcels) which represent a number of real particles with the same properties (i.e. size, velocity and temperature) are calculated. In stationary flows a sequential tracking of the parcels may be adopted, while in unsteady flows all parcels need to be tracked simultaneously on the same time level. Local average properties such as dispersed phase density and velocity are obtained by ensemble averaging. Statistically reliable results for each computational cell require the tracking of typically between 10,000 and 100,000 parcels, depending on the considered flow. The advantage of this method is that physical effects influencing the particle motion, such as particle-turbulence interaction, particle-wall collisions, and collisions between particles can be modelled on the basis of physical principles. Moreover, a particle size distribution may be easily considered by sampling the size of the injected particles from a given distribution function. Problems however may be encountered in the convergence behaviour for high particle concentration due to the influence of the dispersed phase on the fluid flow (i.e. two-way coupling) which is accounted for by

source terms obtained through averaging particle trajectories (Crowe et al. 1977, Kohnen et al. 1994).

Essential for a reliable application of both methods is the appropriate modelling of the relevant physical mechanisms affecting the particle motion, as for example, turbulent transport of particles, wall interactions of particles, collisions between particles and agglomeration. In some cases the physical phenomena are far too complicated to allow for a derivation of the model from basic principles of physics (e.g. particle agglomeration). Therefore, detailed experiments are required to analyse the considered phenomenon and to derive appropriate empirical or semi-empirical models. In order to validate the models, the results of the numerical predictions need to be compared with bench mark test cases featuring the considered phenomenon.

## 7. Nomenclature

|                      |       |                                         |
|----------------------|-------|-----------------------------------------|
| $A_C$                | [-]   | acceleration number                     |
| $C_B$                | [-]   | coefficient of Basset force             |
| $C_{LS}$             | [-]   | lift coefficient for slip-shear         |
| $C_{LR}$             | [-]   | lift coefficient for slip-shear         |
| $C_R$                | [-]   | rotational coefficient                  |
| $C_V$                | [-]   | coefficient of added mass force         |
| $C_D$                | [-]   | drag coefficient                        |
| $C_{D,Stokes}$       | [-]   | Stokes drag coefficient                 |
| $\overline{c_{Mol}}$ | [m/s] | mean relative velocity of gas molecules |
| $D_P$                | [m]   | particle diameter                       |
| $\vec{E}$            | [V/m] | strength of electrical field            |
| $\vec{F}_V$          | [N]   | added mass force                        |
| $\vec{F}_B$          | [N]   | Basset force                            |
| $\vec{F}_e$          | [N]   | electrostatic force                     |
| $\vec{F}_g$          | [N]   | gravity force                           |
| $\vec{F}_i$          | [N]   | forces acting on a particle             |
| $\vec{F}_{LS}$       | [N]   | slip-shear lift force                   |
| $\vec{F}_{LS,Saff}$  | [N]   | Saffman force                           |

|                  |                                   |                                         |
|------------------|-----------------------------------|-----------------------------------------|
| $\vec{F}_{LR}$   | [N]                               | slip-rotation lift force                |
| $\vec{F}_p$      | [N]                               | pressure force                          |
| $\vec{F}_w$      | [N]                               | drag force                              |
| $\vec{g}$        | [m/s <sup>2</sup> ]               | gravity vector                          |
| $I_p$            | [kg m <sup>2</sup> ]              | moment of inertia                       |
| $k$              | [m <sup>2</sup> /s <sup>2</sup> ] | turbulent kinetic energy                |
| $Kn$             | [-]                               | Knudsen number                          |
| $m_p$            | [kg]                              | mass of a particle                      |
| $p$              | [N/m <sup>2</sup> ]               | pressure                                |
| $q_p$            | [C]                               | particle charge                         |
| $R_p$            | [m]                               | particle radius                         |
| $Re_{krit}$      | [-]                               | kritical Reynolds number                |
| $Re_p$           | [-]                               | particle Reynolds number of translation |
| $Re_R$           | [-]                               | particle Reynolds number of rotation    |
| $Re_S$           | [-]                               | particle Reynolds number of shear       |
| $\vec{T}$        | [N m]                             | torque                                  |
| $t$              | [s]                               | time                                    |
| $\vec{u}_F$      | [m/s]                             | instantaneous fluid velocity vector     |
| $\vec{u}_p$      | [m/s]                             | instantaneous particle velocity vector  |
| $\dot{V}$        | [m <sup>3</sup> /s]               | volume flow rate                        |
| $v_r$            | [m/s]                             | radial velocity                         |
| $v_\phi$         | [m/s]                             | tangential velocity                     |
| $\vec{x}_p$      | [m]                               | particle position vector                |
| $\varepsilon$    | [m <sup>2</sup> /s <sup>3</sup> ] | dissipation of turbulent kinetic energy |
| $\lambda$        | [m]                               | mean free path of the gas               |
| $\mu_F$          | [Pa s]                            | dynamic viscosity                       |
| $\rho_F$         | [kg/m <sup>3</sup> ]              | density of the fluid                    |
| $\rho_p$         | [kg/m <sup>3</sup> ]              | density of particle material            |
| $\sigma$         | [m/s]                             | mean fluctuating velocity               |
| $\vec{\omega}_F$ | [1/s]                             | rotation of the fluid                   |
| $\vec{\omega}_p$ | [1/s]                             | angular velocity of the particle        |

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